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STEERING OF ONE VEHICLE GENERAL APPROACH FOR THE PLANE-MODEL

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INTRODUCTION

General case of vehicle's movement should be presented with the model, which includes the movement of vehicles in real conditions, i.e. rectilinear and curvilinear movement, which implies the presence of side forces in the dependence of centrifugal force as well as of longitudinal forces caused by braking or acceleration.

Through specific commands driver affects the size of towing force and braking force, and by means of the steering wheel affects the size of the side force.

In this paper, a two-axle model of passenger vehicles with two traces was analysed.

There are two coordinate systems, a stationary coordinate system OXYZ and a moving coordinate system (oxyz) related to the vehicle's center of gravity, where:

- x-axis is a longitudinal axis of vehicle's center of gravity
- y- axis lateral axis of vehicle's center of gravity
- z- axis vertical axis of vehicle's center of gravity

1. MATHEMATICAL MODEL

Bearing in mind the aim of this paper, so-called output values, a complex model was made shown at Figure 1.

The model has three degrees of freedom:

- x - body's displacement along the x-axis
- y - body's displacement along the y-axis
- ψ - sidesliping, oscillations around the z – axis.

Marks given at Figure 1 are:

- *a, b* – distance of the center of gravity related to the front and rear axle
- $d_1 + d_2$ – trace of the front wheels
- $d_3 + d_3$ – trace of the rear wheels

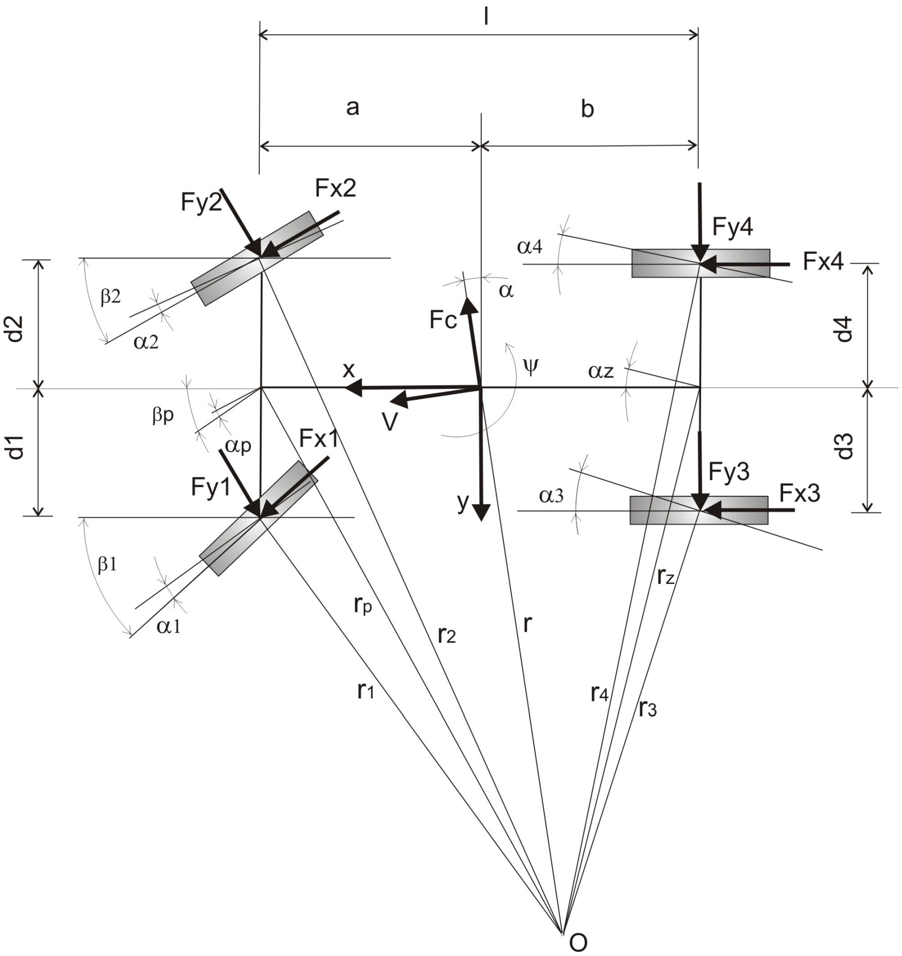

Figure 1: *Model of the vehicle during the turning*
(**r_i, i** = 1, 4 – radiuses of wheel's turning, **r_p, r_z**– radiuses of the front and rear axle mid points turning, **r** – radius of center of gravity turning)

Angles relevant for this model, shown at figure 1, should be defined:

$$\beta = \frac{\beta_1 + \beta_2}{2} \quad (1) \qquad \alpha_1 = \beta_1 - \frac{y' + a \cdot \psi'}{x' - d_1 \cdot \psi'} \quad (2)$$

$$\alpha_2 = \beta_2 - \frac{y' + a \cdot \psi'}{x' + d_2 \cdot \psi'} \quad (3) \qquad y' = \frac{d_y}{d_t};\ x' = \frac{d_x}{d_t}$$

ψ - angle of sidesliping

β - rear turning angle of steering wheels

β_1, β_2 – turning angle of an inner and outer wheel
α_1, α_2 – angles of lateral drifting of front inner and outer wheel
α_3, α_4 – angles of lateral drifting of rear inner and outer wheel
α_p – mid value of steering wheel's turning angle

$$\alpha_3 = -\frac{y' - b \cdot \psi'}{x' + d_3 \cdot \psi'} \quad (4) \qquad \alpha_4 = -\frac{y' - b \cdot \psi'}{x' - d_4 \cdot \psi'} \quad (5)$$

$$\alpha_p = \beta_p - \frac{y' + a \cdot \psi'}{x'} \quad (6) \qquad \alpha_z = -\frac{y' - b \cdot \psi'}{x'} \quad (7)$$

$$\alpha = \frac{y'}{x'} \quad (8) \qquad \alpha = \frac{d_y}{d_x} = \frac{\frac{d_y}{d_t}}{\frac{d_x}{d_t}} = \frac{y'}{x'}$$

Differential equations of vehicle's motion are:

$$m \cdot x'' = F_{1x} \cdot \cos\beta_1 + F_{2x} \cdot \cos\beta_2 - c_{y1} \cdot \left(\beta_1 - \frac{y' + a \cdot \psi'}{x' - d_1 \cdot \psi'}\right) \cdot \sin\beta_1 -$$
$$- c_{y2} \cdot \left(\beta_2 - \frac{y' + a \cdot \psi'}{x' + d_2 \cdot \psi'}\right) \sin\beta_2 + F_{3x} + F_{4x} + F_c \cdot \sin\left(\frac{y'}{x'}\right)$$

$$m \cdot y'' = F_{1x} \cdot \sin\beta_1 + F_{2x} \cdot \sin\beta_2 + c_{y1} \cdot \left(\beta_1 - \frac{y' + a \cdot \psi'}{x' - d_1 \cdot \psi'}\right) \cos\beta_1 +$$
$$+ c_{y2} \cdot \left(\beta_2 - \frac{y' + a \cdot \psi'}{x' + d_2 \cdot \psi'}\right) \cdot \cos\beta_2 + c_{y3} \cdot \alpha_3 + c_{y4} \cdot \alpha_4 - F_c \cdot \cos\left(\frac{y'}{x'}\right) \quad (9)$$

$$J_z \cdot \psi'' =$$
$$\left(F_{1x} \cdot \sin\beta_1 + F_{2x} \cdot \sin\beta_2 + c_{y1} \cdot \left(\beta_1 - \frac{y' + a \cdot \psi'}{x' - d_1 \cdot \psi'}\right) \cos\beta_1 + c_{y2} \cdot \left(\beta_2 - \frac{y' + a \cdot \psi'}{x' + d_2 \cdot \psi'}\right) \cdot \cos\beta_2\right) \cdot a -$$
$$-\left(c_{y3} \cdot \alpha_3 + c_{y4} \cdot \alpha_4\right) \cdot b - F_{1x} \cdot \cos\beta_1 \cdot d_1 + F_{2x} \cdot \cos\beta_2 \cdot d_2 + F_{3x} \cdot d_3 - F_{4x} \cdot d_4 +$$
$$+ c_{y1} \cdot \left(\beta_1 - \frac{y' + a \cdot \psi'}{x' - d_1 \cdot \psi'}\right) \cdot \sin\beta_1 \cdot d_1 - c_{y2} \cdot \left(\beta_2 - \frac{y' + a \cdot \psi'}{x' + d_2 \cdot \psi'}\right) \cdot \sin\beta_2 \cdot d_2$$

Where:

$F_c = \frac{m \cdot v^2}{R}$ - is a centrifugal force (10)

$\alpha = \frac{y'}{x'}$ - is a divergence angle of a supported mass center's speed due to side drifting in relation to the longitudinal axis (angle of sidesliping) (11)

ψ - is a total rotation of supported mass center around the vertical axis.

$c_{yi} \quad i = 1, 4$ is a lateral rigidity of tires.

Lateral force of the wheels is defined with following relations:

$F_{1y} = c_{y1} \cdot \alpha_1$

$F_{2y} = c_{y2} \cdot \alpha_2$

$F_{3y} = c_{y3} \cdot \alpha_3$

$F_{y4} = c_{y4} \cdot \alpha_4$, from which follows: $F_{yi} = c_{yi} \cdot \alpha;\ i = 1 \div 4$ (12)

Longitudinal force might be braking force or pulling force. Total longitudinal force is marked as F_x, while the latitude wheel forces are marked as F_{xi}.

For smaller turning can be applied: $F_p \approx F_{x1} + F_{x2}$ (13),

where as for the rear axle: $F_z = F_{x3} + F_{x4}$ (14)

F_p - front axle ,

F_z - rear axle,

If K_o is a coefficient of distribution of tensile force forward-backward will be:

$F_p = K_o \cdot F_x$, front latitude forces (15)

$F_z = (1 - K_o) \cdot F_x$ (16),

then following relations can be applied:

$K_o = 0 \qquad F_p = 0;\ F_z = F_x$ (17.1)

$0 < K_o < 1 \qquad F_p = K_o \cdot F_x;\ F_z = (1 - K_o) \cdot F_x$ (17.2)

$K_o = 1 \qquad F_p = F_x;\ F_z = 0$ (17.3)

Distribution of forces on the front wheels, left/right on the front axle:

$0 \le K_{tp} \le 1 \qquad F_{x1} = K_{tp} \cdot F_{px} \qquad F_{2x} = (1 - K_{tp}) \cdot F_{px}$

Distribution of axle forces on the rear wheels, left/right on the rear axle:

$0 \le K_{tz} \le 1 \qquad F_{3x} = K_{tz} \cdot F_{zx} \qquad F_{4x} = (1 - K_{tz}) \cdot F_{zx}$

2. RESULTS OF COMPUTING

Example given in this paper is conducted a case of realignment of vehicles

INPUT DATA

a =1.2 m;	l =2.5 m;	d_i =0.7 m;	i = 1,2,3,4
Jz=2000 kgm^2;	m=1000 kg;	v=10 m/s;	R=20 m

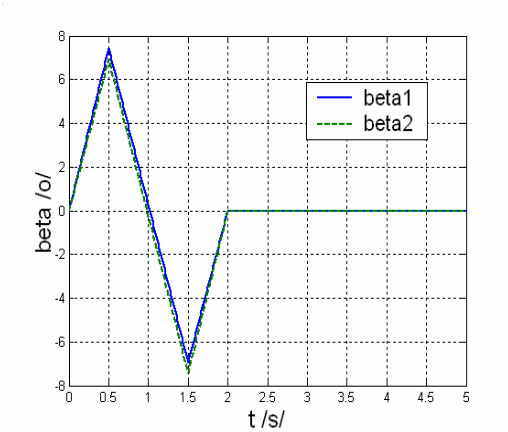

Figure 2: *Angle of the steering wheel rotation*

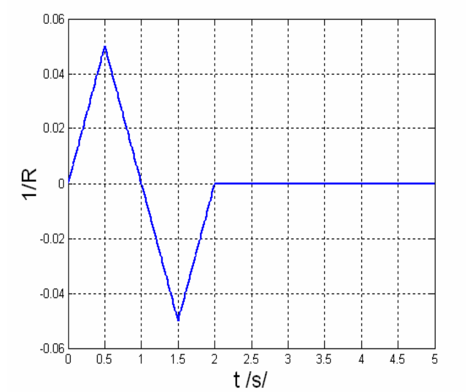

Figure 3: *Curvature*

OUTPUT RESULTS

On the following Figures are presented the result of this computing.

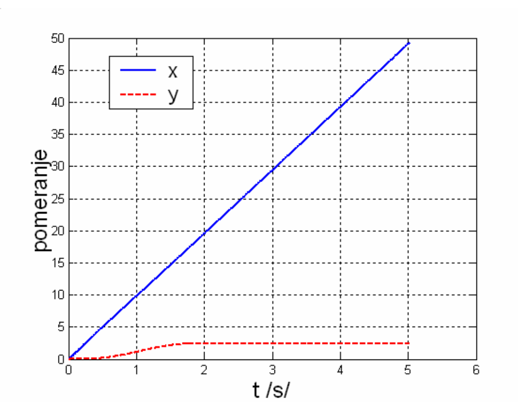

Figure 4: *Vehicle's center displacement along the x and y axis*

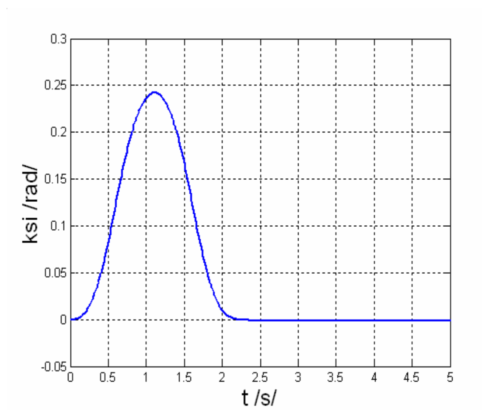

Figure 5: *Angle of the vehicle's rotation around the vertical axis*

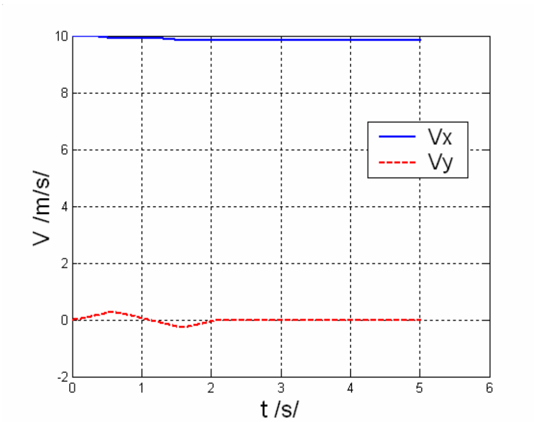

Figure 6: *Speed of the vehicle's center displacement along x and y axi*

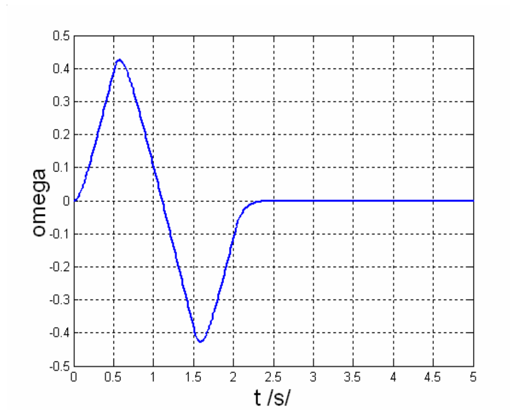

Figure 7: *Angular speed of the vehicle's center rotation around the vertical axis*

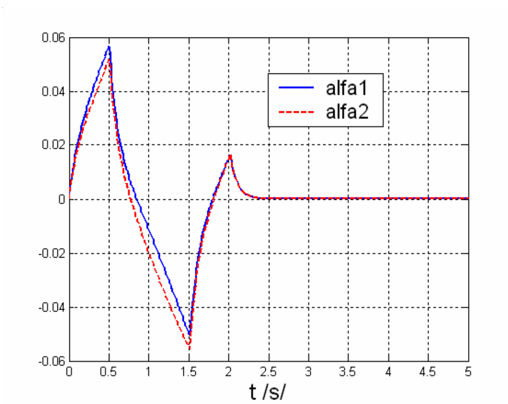

Figure 8: *Angle of the front wheel's rotation*

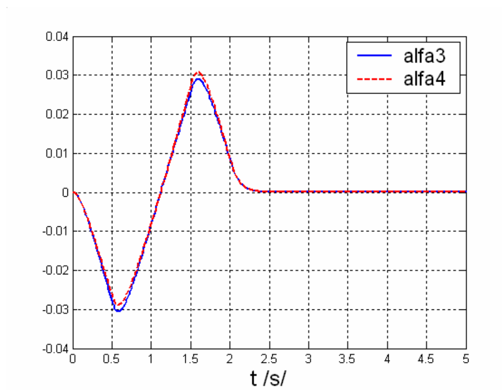

Figure 9: *Angle of the rear wheel's rotation*

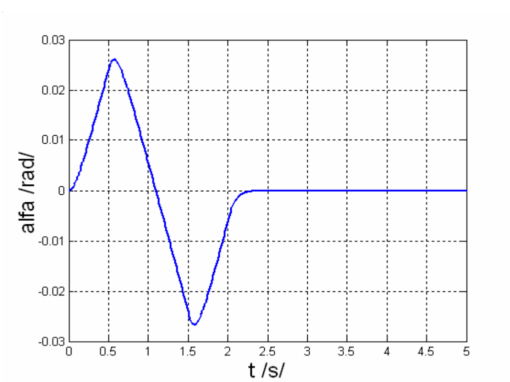

Figure 10: *Angle of the vehicle's center rotation*

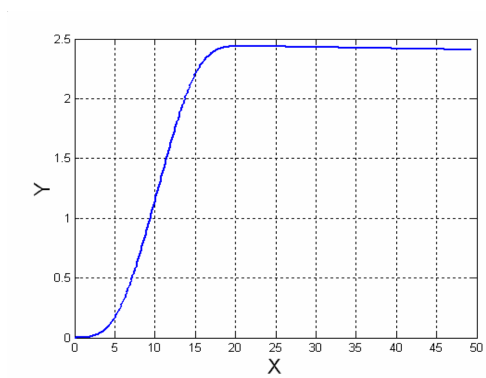

Figure 11: *The path of the vehicle at the given movement*

CONCLUSION

Dynamic equations provide the possibility of thoroughly analysis of vehicle's movement in different regimes (rectilinear drive, maneuvers, turnovers, movement along the curvatures). Variation of vehicle's maneuverability can be monitored in different regimes of movement (braking or accelerating in the curvatures, which is extremely important from the safety aspect). This analysis is especially important during traffic accident analysis.

Moreover, with alteration

- of the mass distribution and the gravity center position, an influence of the axle loading to the stability
- of the forces F_X, braking -acceleration regime
- of the side forces F_y (F_{yi}), regime of drive along the curvatures
- of the characteristics of tires Cyi, a tire's state or type of the tires

can be analysed.

Entering these equations in any simulation software provides the possibility of analysis of many problems related to dynamical vehicle's values, which are reflected to the vehicles safety.

ACKNOWLEDGMENT

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REFERENCES

[1] Janković A.:Vehicle's dynamics, Faculty of Mechanical Engineering, Kragujevac, 2008.

[2] Matschinsky W.: Die Radfuhrungen der Strabenfahryeuge, Werlag TUV Rheinland, 1987.

[3] Radonjić R.: Identifikacija dinamičkih karakteristika motornih vozila, Faculty of Mechanical Engineering, Kragujevac, 1995.

[4] Poolone G.: Construzioni automobilistiche il veicolo, Torino, 1970.

[5] Remling J.: Breakes, John Wiley@Sons, New York, 1978.

[6] Reimpell J.: Fahrwerktechnik 1, Woge Werlage, 1971.

[7] Reimpell J.: Fahrwerktechnik 2, Woge Werlage, 1973.

[8] Janković A., Lalović D., Đukić M.: Suspension of passengers' vehicles (in serbian Oslanjanje putničkog vozila), R&D Institute "Zastava - auto", Kragujevac, 1998.