THE DYNAMIC ANALYSIS OF A VEHICLE'S MOTION AT THE POINT OF CORNERING

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INTRODUCTION

A vehicle's circular motion entails a kind of motion which is performed by a vehicle along the circle of a specific radius all the way to the point of cornering which corresponds to the real conditions, with the presence of lateral forces depending on the centrifugal force and longitudinal forces in the form of braking and accelerating.

Apart from this, a significant influence is exerted by a driver himself / herself, because he / she is the one who alters the magnitude and character of external forces which have an influence upon a motor vehicle. The magnitudes of a driving force and braking force are also under the influence of a driver's use of specific controls, and it is the steering wheel that enables a driver to maintain the circular course of a vehicle's motion.

What is being examined in this work is the spatial model of a medium class passenger vehicle of a conventional construction which is used for observing the parameters, on the basis of which a vehicle's behaviour is defined depending on the forces resulting from the circular motion of a particular vehicle.

The two axis systems have been introduced: a system of earth – fixed axis and the one with non – fixed axis, which is related to a vehicle's position oxyz where:

the X – axis represents the lateral axis of a vehicle's gravity center;

the Y – axis represents the longitudinal axis of a vehicle's gravity center and

the Z – axis represents the yaw axis of the gravity center.

1. A MATHEMATICAL MODEL

Taking into account the aim of this work, what has been done – is a complex model of a vehicle which enables the following parameters to be analyzed:

x – a body displacement along the x – axis,

y – a body displacement along the y – axis,

ψ – a sideslip, oscillations about the x – axis,

a, b – a vehicle wheelbase,

h – the height of center of gravity and

2s – a wheel track.

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The Figure 1 represents the vertical dynamics of a vehicle provided with the analysis of forces and displacements depending on the center of gravity displacement.

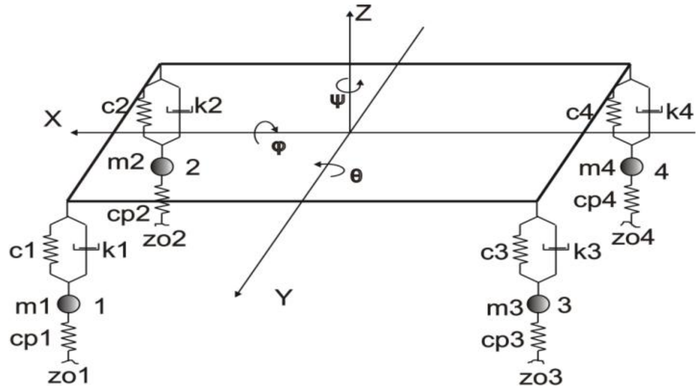

Figure 1: *The vertical dynamics of a vehicle*

The Figure 2 represents the horizontal dynamics of a vehicle while cornering provided with the analysis of forces and displacements.

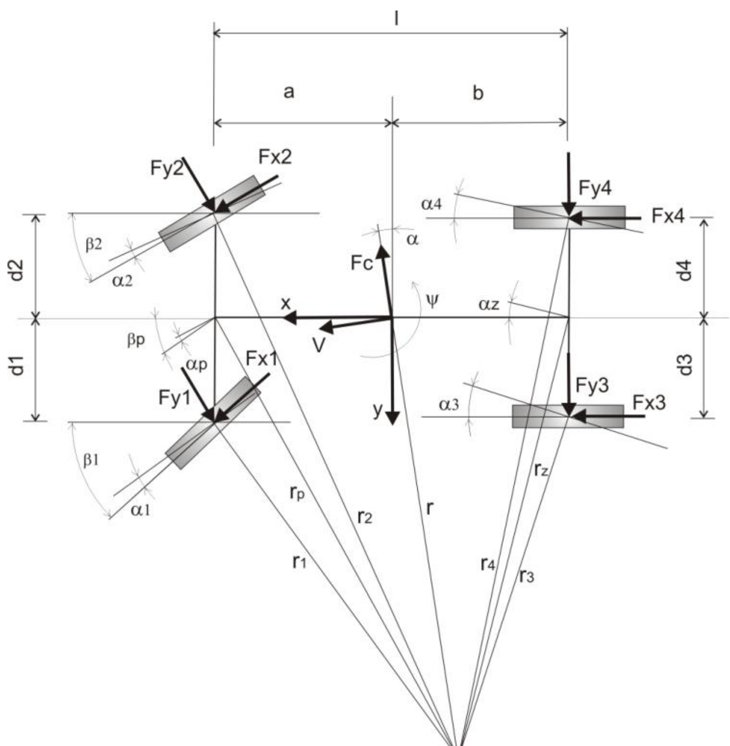

Figure 2: *The horizontal dynamics of a vehicle*

The angles relevant for the horizontal dynamics as shown in the Figure 2 – are to be defined in the following manner:

$$\beta = \frac{\beta_1 + \beta_2}{2} \qquad \alpha_1 = \beta_1 - \frac{y' + a \cdot \psi'}{x' - d_1 \cdot \psi'} \qquad \alpha_2 = \beta_2 - \frac{y' + a \cdot \psi'}{x' + d_2 \cdot \psi'}$$

$$\alpha_3 = -\frac{y' - b \cdot \psi'}{x' + d_3 \cdot \psi'} \qquad \alpha_4 = -\frac{y' - b \cdot \psi'}{x' - d_4 \cdot \psi'} \qquad \alpha_p = \beta_p - \frac{y' + a \cdot \psi'}{x'}$$

$$\alpha_z = -\frac{y' - b \cdot \psi'}{x'} \qquad \alpha = \frac{y'}{x'}$$

Differential equations of a vehicle's motion are the following:

$$m_1 \ddot{z}_1 = c_1 (z_5 + a\theta - s\varphi - z_1) + k_1 (\dot{z}_5 + a\dot{\theta} - s\dot{\varphi} - \dot{z}_1) - c_{p1z}(z_1 - z_{01})$$
$$m_2 \ddot{z}_2 = c_2 (z_5 + a\theta + s\varphi - z_2) + k_2 (\dot{z}_5 + a\dot{\theta} + s\dot{\varphi} - \dot{z}_1) - c_{p2z}(z_2 - z_{02})$$
$$m_3 \ddot{z}_3 = c_3 (z_5 - b\theta - s\varphi - z_3) + k_3 (\dot{z}_5 - b\dot{\theta} - s\dot{\varphi} - \dot{z}_3) - c_{p3z}(z_3 - z_{03})$$
$$m_4 \ddot{z}_4 = c_4 (z_5 - b\theta + s\varphi - z_4) + k_4 (\dot{z}_5 - b\dot{\theta} + s\dot{\varphi} - \dot{z}_4) - c_{p4z}(z_4 - z_{04})$$

$$m_5 \ddot{z}_5 = -(c_1 + c_2 + c_3 + c_4) z_5 - ((c_1 + c_2)a - (c_3 + c_4)b)\theta - (c_1 + c_4 - c_1 - c_3) s\varphi +$$
$$+ c_1 z_1 + c_2 z_2 + c_3 z_3 + c_4 z_4 - (k_1 + k_2 + k_3 + k_4)\dot{z}_5 - ((k_1 + k_2)a - (k_3 + k_4)b)\dot{\theta} -$$
$$-(k_2 + k_4 - k_1 - k_3) s\dot{\varphi} + k_1 \dot{z}_1 + k_2 \dot{z}_2 + k_3 \dot{z}_3 + k_4 \dot{z}_4$$

$$I_x \quad \ddot{\varphi} = (c_1 - c_2 + c_3 - c_4) z_5 \quad s +$$
$$+(a \quad c_1 - a \quad c_2 - b \quad c_3 + b \quad c_4)\theta \quad s - (c_1 + c_2 + c_3 + c_4) \quad s^2 \varphi +$$
$$+(-c_1 \quad z_1 + c_2 \quad z_2 - c_3 \quad z_3 + c_4 z_4) s +$$
$$+(k_1 - k_2 + k_3 - k_4) \quad \dot{z}_5 \quad s + ((k_1 - k_2) \quad a - (k_3 - k_4) \quad b) \quad \dot{\theta} \quad s -$$
$$-(k_1 + k_2 + k_3 + k_4) \quad s^2 \dot{\varphi} + (-k_1 \dot{z}_1 + k_2 \dot{z}_2 - k_3 \dot{z}_3 + k_4 \dot{z}_4) s$$

$$I_y\ddot{\theta} = z_5(-a(c_1+c_2)+b(c_3+c_4))+((c_1+c_2)a^2-(c_3+c_4)b^2)\quad \theta +$$
$$+((-c_1+c_2)sa+(-c_3+c_4)\quad sb)\quad \varphi$$
$$a(c_1\quad z_1+c_2\quad z_2)+b(c_3\quad z_3+c_4 z_4)+$$
$$+(a(k_1+k_2)+b(k_3+k_4))\quad \dot{z}_5+$$
$$+((k_1+k_2)\quad a^2-(k_3+k_4)\quad b^2)\quad \dot{\theta}+$$
$$+(\ (-k_1+k_2)s\quad a+(-k_3+k_4)s\quad b)\quad \dot{\varphi}-(a(k_1\dot{z}_1+k_2\dot{z}_2)+b(k_3\dot{z}_3+k_4\dot{z}_4)$$

$$m\cdot x'' = F_{1x}\cdot\cos\beta_1 + F_{2x}\cdot\cos\beta_2 - c_{y1}\cdot\left(\beta_1 - \frac{y'+a\cdot\psi'}{x'-d_1\cdot\psi'}\right)\cdot\sin\beta_1 -$$
$$-c_{y2}\cdot\left(\beta_2 - \frac{y'+a\cdot\psi'}{x'+d_2\cdot\psi'}\right)\sin\beta_2 + F_{3x} + F_{4x} + F_c\cdot\sin\left(\frac{y'}{x'}\right)$$

$$m\cdot y'' = F_{1x}\cdot\sin\beta_1 + F_{2x}\cdot\sin\beta_2 + c_{y1}\cdot\left(\beta_1 - \frac{y'+a\cdot\psi'}{x'-d_1\cdot\psi'}\right)\cos\beta_1 +$$
$$+c_{y2}\cdot\left(\beta_2 - \frac{y'+a\cdot\psi'}{x'+d_2\cdot\psi'}\right)\cdot\cos\beta_2 + c_{y3}\cdot\alpha_3 + c_{y4}\cdot\alpha_4 - F_c\cdot\cos\left(\frac{y'}{x'}\right)$$

$$J_z\cdot\psi'' = \begin{pmatrix} F_{1x}\cdot\sin\beta_1 + F_{2x}\cdot\sin\beta_2 + c_{y1}\cdot\left(\beta_1 - \frac{y'+a\cdot\psi'}{x'-d_1\cdot\psi'}\right)\cos\beta_1 + \\ +c_{y2}\cdot\left(\beta_2 - \frac{y'+a\cdot\psi'}{x'+d_2\cdot\psi'}\right)\cdot\cos\beta_2 \end{pmatrix}\cdot a -$$
$$-\left(c_{y3}\cdot\alpha_3 + c_{y4}\cdot\alpha_4\right)\cdot b - F_{1x}\cdot\cos\beta_1\cdot d_1 + F_{2x}\cdot\cos\beta_2\cdot d_2 + F_{3x}\cdot d_3 - F_{4x}\cdot d_4 +$$
$$+c_{y1}\cdot\left(\beta_1 - \frac{y'+a\cdot\psi'}{x'-d_1\cdot\psi'}\right)\cdot\sin\beta_1\cdot d_1 - c_{y2}\cdot\left(\beta_2 - \frac{y'+a\cdot\psi'}{x'+d_2\cdot\psi'}\right)\cdot\sin\beta_2\cdot d_2$$

$$F_c = \frac{m\cdot v^2}{R} \qquad\qquad \alpha = \frac{y'}{x'}$$

$c_{yi}\quad i=1,4$ cornering stiffness.

A lateral force:

$$F_{1y} = c_{y1}\cdot\alpha_1 \qquad F_{2y} = c_{y2}\cdot\alpha_2 \qquad F_{3y} = c_{y3}\cdot\alpha_3 \qquad F_{4y} = c_{y4}\cdot\alpha_4$$

A longitudinal force can be expressed as a braking force or driving force. It can be displayed through the axle coefficient K , the sign of which can be positive or negative.

F_p - a front axle,

F_z - a rear axle and

F_x - a resultant longitudinal force.

$$F_p = K_o \cdot F_x \qquad F_z = (1 - K_o) \cdot F_x$$

where:

$$K_o = 0 \qquad F_{px} = 0; \; F_{zx} = F_x$$
$$0 < K_o < 1 \qquad F_{px} = K_o \cdot F_x; \; F_{zx} = (1 - K_o) \cdot F_x$$
$$K_o = 1 \qquad F_{px} = F_x; \; F_{zx} = 0$$

A front wheel distribution:

$$0 \le K_{tp} \le 1 \qquad F_{1x} = K_{tp} \cdot F_{px} \qquad F_{2x} = (1 - K_o) \cdot F_{px}$$

A rear wheel distribution:

$$0 \le K_{tz} \le 1$$
$$F_{3x} = K_{tz} \cdot F_{zx}$$
$$F_{4x} = (1 - K_{tz}) \cdot F_{zx}$$

2. THE CALCULATION RESULTS

INPUT DATA

m1=25; m2=25; m3=25; m4=25; m=1000;

c1=42000; c2=42000; c3=60000; c4=60000;
cp1z=170000; cp2z=170000; cp3z=170000; cp4z=170000;
c1bocno=600; c2bocno=600; c3bocno=600; c4bocno=600;

k1=42000; k2=42000; k3=42000; k4=42000;

a=1.2; b=1.3; s=0.7;   d1=s; d2=s;d3=s;d4=s; l=2.5; h=0.6;

Ix=1680;   Iy=570;   Iz=2000;

m=1000 kg;
v=10 m/s;
R=20 m

Input for circular driving

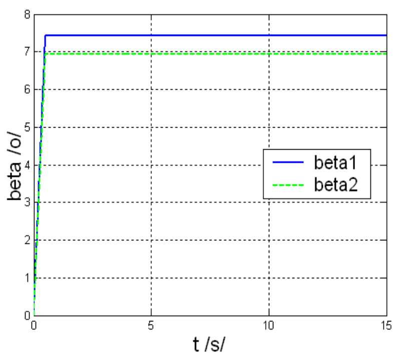

***Figure 3**: The slip angles of front wheels*

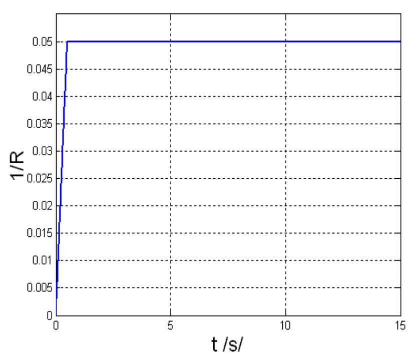

***Figure 4:** The reciprocal value of the corner radius*

OUTPUT RESULTS

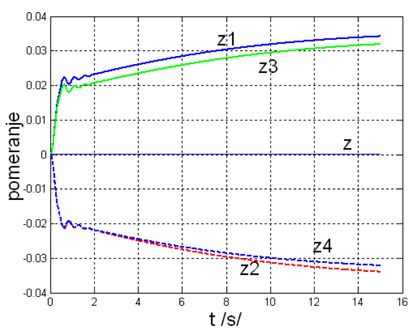

***Figure 5:** A vertical wheel displacement*
z1 – the front left – hand drive wheel
z2 – the front right – hand drive wheel
z3 – the rear left – hand drive wheel
z4 – the rear right – hand drive wheel

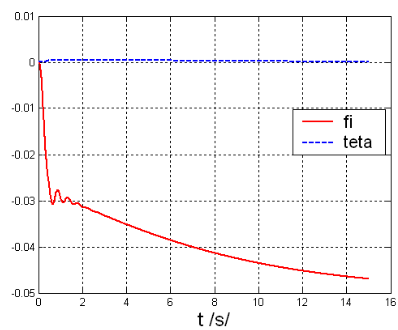

***Figure 6:** The rotation of the sprung mass center of gravity*
fi – around the longitudinal axis
teta – around the lateral axis

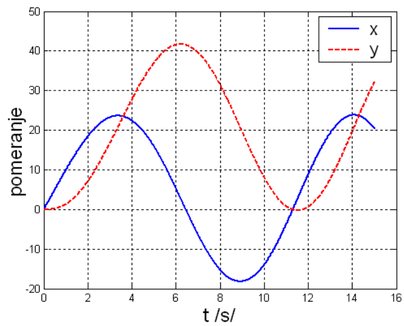

Figure 7: *The displacement of a vehicle's center of gravity in the lateral (y) direction and longitudinal (x) direction*

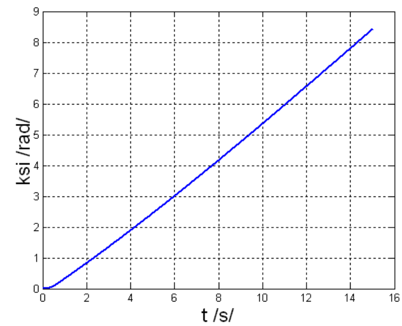

Figure 8: *The angle of the vehicle's rotation around the vertical axis*

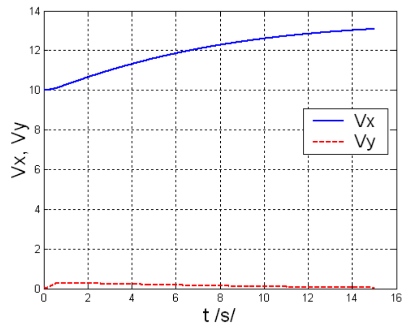

Figure 9: *The velocity of the center of gravity displacement in the lateral (Vy) and longitudinal (Vx) direction*

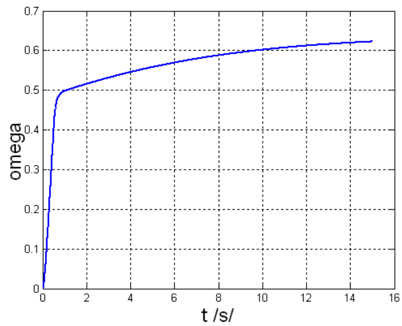

Figure 10: *The angular velocity of the rotations around the vertical axis*

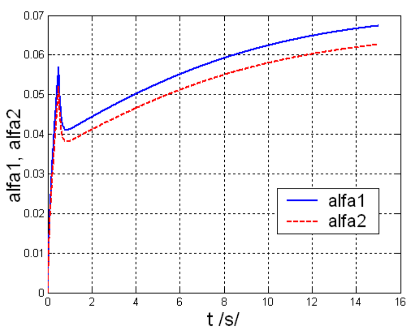

Figure 11: *The lateral slip angles of front wheels*

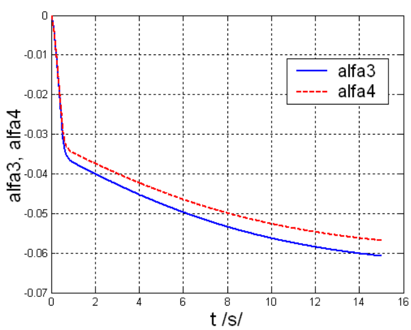

Figure 12: *The lateral slip angles of rear wheels*

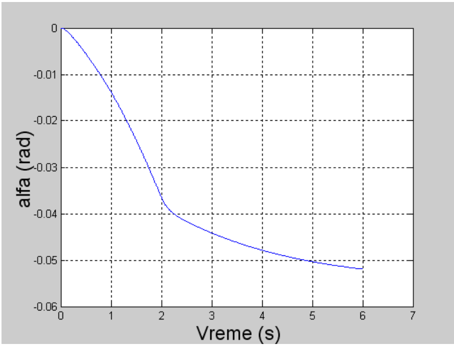

__Figure 13:__ The angle of the sprung mass motion with respect to the longitudinal axis of a vehicle

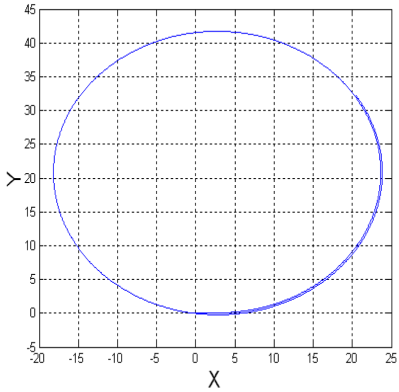

__Figure 14:__ The motion path of a vehicle's center of gravity

CONCLUSION

Dynamics equations presented in this paper provide an analysis of a vehicle's movement along the curvature. The spatial model includes the translations and angular displacements. The model is compared with the well-known Reimpell's models, which are similar to this one.

The results obtained using numerical simulation methods and techniques are presented in a digital and graphical form and provided to make a conclusion about stability and control.

The object of our research was the domestic vehicle, the experimental car where we change the tyres characteristics and load of the vehicle. This model verifies the experimental research made in the previous papers and experimental investigation on road as regards the same vehicle.

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