# **VEHICLE STEERING CONTROL**

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### INTRODUCTION

The wheeled motor vehicle do not possesses own stability of lateral and angular motion along desired path. From this reason, some form of driver steering control or other controller must be included to vehicle stabilization and guidance. Many researchs have been made to study of driver behaviour by different control tasks in different driving conditions, [1], [12]. Numerous mathematical models developed to describe of driver control action are named as, ideal, quasi-linear, optimal, predictive, supervisory and so on. [2], [3], [4], [13], [14]. The term driver model is a synonym for a mathematical description driver control with models of vehicle dynamics in interaction with road and environment.

Driver models were developed, before all, in order to get information about how a vehicle and its design changes influence the handling quality and driver efforts, also to accident causations. Recently, theoretical and experimental researchs of driver – vehicle – environment interaction have great importance with aspect to above pointed out problems but with aspect design, optimisation and application automated vehicle control, [9]. According to above mentioned problems in this paper some phenomenon of vehicle unstability are studied and any solution to assessment vehicle steering performances are proposed, in order to continue our earlier researchs, [8], [15].

### **1. VEHICLE LATERAL DYNAMICS**

The vehicle lateral dynamics can be modelled with different complexity, from a simple two – wheeled model to a numerous degree of freedom model. The complex vehicle model requires using of numerical integration which makes it difficult to form general conclusion. In many currently researches developed vehicle steering controllers for automated vehicles have focused on low lateral acceleration conditions, [9]. Under these conditions linear vehicle and tire models are suitable for the controllers developing and design. However, most emergency situation require complex vehicle model, which today can be successfully realized by means of advanced simulation methods, [10].

In this paper, a simplified model is used, which simulates vehicle motion over a flat and level road surface, when the forward speed is kept constant. The equations of motion are written in relation to vehicle lateral  $\Delta y$ , and heading deviation  $\Delta \varepsilon$ , from desired path, which represent absolute lateral displacement y, and heading angle  $\varepsilon$ , on the straight-line road, according to presentation in Fig. 1a, vehicle model and Fig. 1b, relevant inputs to vehicle steering control. As can see from implicit expressions (1) and (2), output variables, y and  $\varepsilon$ .

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are mutually coupled by means of the system matrices coefficients,  $a_{ji}$ ,  $b_j$ . Input variable is steering wheel angle, denoted with,  $\beta$ . The initial equations (1) and (2) are transformed into state space form suitable to control problems solution, (3), (4), (5).

Equation (3), as matrix expression, presents state space form of basic vehicle model in open-loop with state vector (4). On the other hand, matrix equation (5), describes closed loop system in state space form for alternative control defined by state vectors: (5.1) - driver steerig control, (5.2) - optimal controller application, (5.3) - combined control, driver action supproted by technical controller for given condition, so called vehicle stability augmentation systems, which modifies the driver steering command, as presented in Fig. 2, internal loop with input variables, *y*,  $\varepsilon$ , and output variable,  $\beta_2$ .



Figure 1a: Model of vehicle lateral dynamics



*Figure 1b: Real and estimated inputs to vehicle steering control, according to Fig. 1a, a/ driver, b/ optimal controller* 

$$\ddot{\mathbf{y}} = \ddot{\mathbf{y}}(a_{22}\dot{\mathbf{y}}, a_{23}\varepsilon, a_{24}\dot{\varepsilon}, b_2) \tag{1}$$

$$\ddot{\varepsilon} = \ddot{\varepsilon}(a_{42}, \dot{y}, a_{43}\varepsilon, a_{44}\dot{\varepsilon}, b_4) \tag{2}$$

$$\dot{x}_1 = Ax_1 + Bu_1 \tag{3}$$



Figure 2: Possible variants of vehicle steering control

$$x_1 = [y, \dot{y}, \varepsilon, \dot{\varepsilon}] \tag{4}$$

$$u_1 = [\beta] \tag{5}$$

$$\dot{x}_{\mathcal{C}} = Ax_{\mathcal{C}} + Bu_{\mathcal{C}} \tag{6}$$

$$x_{\mathcal{C}} = [y, \dot{y}, \varepsilon, \dot{\varepsilon}, \beta(\dots)] \tag{7}$$

$$\beta_{CV} = \beta_{CV}(y, \varepsilon, \alpha, L) \tag{7.1}$$

$$\beta_{CO} = \beta_{CO}(y, \dot{y}, \varepsilon, \dot{\varepsilon}, ....) \tag{7.2}$$

$$\beta_{cc} = \beta_{cc} \left( \beta_{cv}, \beta_{co} \right) \tag{7.3}$$

$$u_c = [y_0(\varepsilon_0)] \tag{7.4}$$

The more attention in this paper is dedicated to design an optimal controller to vehicle steering control according to theoretical considerations given in chapter number 3, but with previous hypothetic coments about possible driver steering control strategies given in next chapter, 2.

### 2. DRIVER STEERING CONTROL STRATEGIES

Driver control action by a single – loop compensatory task can be described by means of differential equation derived from driver convencional quasi – linear model, [2], [5], [8], and presentation in Fig. 1b, as relationship between steering wheel movement,  $\beta$  and vehicle lateral deviation,  $\Delta y$ :

$$\ddot{\beta}[(\tau+T_N)T_i] + \dot{\beta}(\tau+T_N+T_i) + \beta = -KT_L \Delta \dot{y} - K\Delta y$$
(8)

where,  $\tau$  - driver time delay,  $T_N$  - driver neuromuscular system time lag,  $T_L$ ,  $T_i$  - driver time lead and time lag equalization, respectively, K - driver gain factor.

Driver control strategy in a comprising multi – loop system, shows in Fig. 3, is based on the combined control of the lateral deviation,  $\Delta y$ , into outer loop, and heading deviation  $\Delta \varepsilon$ , into internal loop. So, driver compensate directly lateral deviation and indirectly angular deviation based on the modification of his mode control in outer loop.

By two level control strategies, [6], [7], [8], driver used visual cues of sight field derived from vehicle position relative to road, for compensatory action in equvalent single – loop, advanced in time, and parameters of reference path, as curvature, focused curve segments and so on , to guidance tasks. These three driver strategies, hier presented , can help by design optimal controller and by interpretation results in next chapter.



Figure 3: Driver steering control strategy in multi-loop systems

#### 3. DESIGN OF VEHICLE STEERING CONTROLLER

The vehicle optimal controller has been designed using linear quadratic regulator method based on the optimal control theory, [11]. Therefore, some basic design phase and ideas of the theory should be presented. The dynamic equations of the closed loop system in Figure 4 are present in a state space form:

$$x = Ax + Bu \tag{9}$$

where,  $x = [x_1 \ x_2 \ ... x_n]$ , is the state vector in general form,  $u = [u_1 \ u_2 \ ... u_m]$ , the input vector determined by controller. The output vector, y, can be defined in different forms as a linear combination of the state variables:

$$y = Cx \tag{10}$$

depending on the chosen form of matrix C. The optimal control concept is formulated as a problem to minimize a functional of general form:

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$$J = \frac{1}{2} y^{T}(t_{f}) Sy(t_{f}) + \frac{1}{2} \int_{t_{o}}^{t_{f}} \left[ y^{T}(t)Q(t)y(t) + u^{T}(t)R(t)u(t) \right] dt$$
(11)

where Q, R, S are symmetric weighting matrices, u(t) input, y(t) output variables. With Hamilton – Jacobi equation

and by used

$$H[y(t), u(t), \lambda(t), t] = \frac{1}{2} y^T Q y + \frac{1}{2} u^T R u + \lambda^T A y + \lambda^T B u$$
(12)

maximum principle:

$$\frac{\partial H}{\partial u} = 0 = R(t)u(t) + B^{T}(t)\lambda(t)$$

$$\frac{\partial H}{\partial y} = -\dot{\lambda} = Q(t)y(t) + A^{T}(t)\lambda(t)$$
(13)

at constraint condition:

$$\lambda(t_f) = \frac{\partial \theta}{\partial y(t_f)} = Sy(t_f)$$
(14)

needed solution of (9) is :

$$u(t) = -R^{-1}(t)B^{T}(t)\lambda(t)$$
(15)

with presumed partial solution:

$$\lambda(t) = P(t)y(t) \tag{16}$$

where P(t) is the solution of Riccati matrix equation:

$$\dot{P} = -P(t)A(t) - A^{T}(t)P(t) + P(t)B(t)R^{-1}(t)B^{T}(t)P(t) - Q(t)$$
(17)

by constraint condition  $P(t_f) = S$ .

With P(t) solution from equation (13) can be synthesized optimal controller structure in time domain:

$$u(t) = K(t)y(t) = -R^{-1}(t)B^{T}(t)P(t)y(t)$$
(18)

In this paper optimization control problem is solved with respect to algorithm from equation (9) to equation (18) for defined control task variants in internal loop,on the Fig 2. The results of the Riccati matrix equation solutions are used of line in simulation procedure. Vehicle handling characteristics without control and with optimal controller was idetified in simulation procedure for input data a typical passenger car and some results presented in next chapter.

# 4. RESULTS

Simulation results for typical passenger car are shown in Figures 4 to 25. Possible simulation variables are, system structure, matrix format C as relation system outputs and system state, format and values of weighting matrices, S, Q, R, denoted in previous chapter, also, optimization criterion, further, forward vehicle velocity, different combination of controller/vehicle system functional and design parameters, shape of reference path on the roadway, etc. On the presented Figures in this chapter, are used following variables name or/and signs: latac – lateral acceleration, y'', latvel – lateral velocity, y', latdev – lateral deviation, y, yawac – yawing acceleration,  $\varepsilon''$ , yawvel – yawing velocity,  $\varepsilon'$ , yawan – yawing angle,  $\varepsilon$ .

The results in Figure 4, 5, 6, 7, present vehicle behaviour by fixed steering wheel, initial lateral velocity of 5m/s as vehicle disturbance of initial conditions, longitudinal vehicle velocity, 20 m/s. In Fig. 8, are compared the change of variables from Fig. 4, 5, 6, 7, but by lateral disturbance of 3m/s, and longitudinal velocity 10 km/h. At same condition, Fig. 9, shows time change of vehicle lateral deviation by tunning steering wheel according sinus change with 0.1 rad magnitude and 0.1 Hz frequency.

Figure 10, 11, 12, 13, show vehicle behaviour by impulse disturbance on the front steering wheels, at a constant speed of 20 m/s, in a straight-line path.

The influence of the optimal controller on the vehicle steering in straight-line direction for initial condition disturbance type, a/ initial lateral displacement of 1m, and b/ initial lateral velocity of 5 m/s, are presented in Figures 8 and 9, respectively.

The vehicle time response to lateral disturbance, presented in Fig. 4, 8, as well as, to sinus and impulse disturbance on the steering wheel, presented in Fig. 9 and 10,, illustrate examples of unstable motion releted to lateral motion. Namely, after an instantaneous disturbance at zero initial time the vehicle lateral deviation, shorter name " latdev", presented in mentioned Figures, increases incessantly with time. Similar behaviour vehicle exhibits releted to angular deviation, as presented in Fig. 5, 8, 11, but with different deviation curve slope and settling time.





Figure 4: Vehicle lateral deviation versus time

Figure 5: Vehicle angular deviation versus time





Figure 6: Vehicle lateral acceleration versus time



*Figure 8:* Variables from Fig. 4–7, at lateral disturbance of 3m/s, longitudinal velocity of 5m/s



Figure 7: Vehicle yaw acceleration versus time



Figure 9: Lateral deviation versus time at sinus tunning steering wheel. Lateral disturbance of 3m/s, longitudinal velocity 5m/s



*time time* 

Figure 11: Vehicle angular deviation versus time

Impulse disturbance on steering wheel, longitudinal velocity 20m/s.







The simulation results in Figs. 14 - 25, present vehicle behaviour in the case when optimal controller is acting. The optimal controller structure is determined according to described mathematical relations (9) to (18) with full state vector used as output vector to be minimized. Fig. 14 - 19, show effects optimal controller by initial lateral displacement disturbance of 1m at initial zero time. During optimal controller action, this displacement as lateral deviation from desired path is corrected and vehicle is returned back to its straight-line steady cruise condition.



Figure14: Lateral deviation versus tim.

Optimal controller, lateral disturbance 1m.



Figure 15: Lateral velocity versus time



Figure16: Lateral acceleration versus time

Optimal controller, lateral disturbance 1m.





Picture17: Angular deviation versus time



Figure 20: Lateral deviation versus time

Optimal controller, lateral disturbance 5m/s.



Figure 22: Lateral acceleration versus time





Figure21: Lateral velocity versus time



Figure 23: Aangulardeviation versus time



Figure 25: Yaw acceleration versus time

In Figure 14 - 19, the lateral deviation, latdev, is a well damped process, with relatively shorter settling time, but time response of the lateral velocity and yawing angle shown any sharper peaks, unsymmetrical in relation to zero levels. On the other hand, lateral acceleration, yawing acceleration and yawing velocity are symmetrical well damped processes. These effects can be influenced to vehicle ride quality and reduced by means an appropriate choice of the performance index in optimization procedure, equation (11).

In Figure 20 - 25, results of the vehicle optimal control by lateral disturbance of 5m/s are presented. In this case, control process of the lateral deviation is more difficult task, while the rest time responses are symmetrical damped.

# 5. CONCLUSIONS

The problem of vehicle directional unstability play important role by design new as well as by improvment existing vehicle types. The useful solutions can be obtained by consideration of vehicle as part of closed-loop system with various variants of steering control : (a) by driver, (b) by technical controller, (c) by driver and technical controller simultaneously. The previous studies of driver used control strategies give a good basis to better understanding requirements by choice structure and parameters of optimal controller. On the other hand, possible driver strategies and estimated controller design concept can help to vehicle optimal design according to, today all severer of traffic safety requirements. The proposed methodology and obtained results in this paper can contribute in this direction, before all, to assessment of vehicle handling performance quality with respect to above mentioned requirements and driver effort to steer vehicle.

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