COMPARATIVE ANALYSIS ON MATHEMATICAL MODELS DESCRIBING VIBRATIONS OF AUTOMOTIVE INDEPENDENT SUSPENSIONS

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INTRODUCTION

Kinematical structure of the vehicle independent suspension with increased speeds of motion have been refined over time and today there are a large number of its variants - single, double, and more arm suspensions. Underlying all these are single arm suspensions [6].

The aim of this work is using the methods of vector mechanics to analyze the results for both types of suspensions for the generalization of the computing process, which form the basis of an automated computer program to select the elastic characteristics of the suspension. Numerical experiments are conduct with MATLAB.

THREE-DIMENSIONAL MATHEMATICAL MODELS OF ARM SUSPENSIONS DESCRIBING THE SMOOTHNESS OF MOTION WITH THE METHODS OF VECTOR MECHANICS

The most accurate description behavior of the vehicle is achieved by using three-dimensional mathematical models. The advantage of such schemes is that it is possible to investigate the relocation of the car and turns along the axes O_x, O_y and O_z of the coordinate system located in the center of gravity (i.e. all degrees of freedom) which is a premise for high accuracy in computation process [7]. Schemes of the models are shown in Figure 1 and Figure 2.

The systems under consideration consists suspended and nosuspended masses. The suspended masses include the masses of the elements of the car body, passengers and load. In the center of gravity is fixed local coordinate system O_0x_0y_0z_0. The suspension is implemented as a tire, arm, axle and other components are combined in one element which is hinged to the suspended masses [10].

Each of these elements is fixed to local coordinate system, respectively O_1x_1y_1z_1, O_2x_2y_2z_2, O_3x_3y_3z_3, O_4x_4y_4z_4. In the equilibrium position the axis of the all coordinate systems are parallel. All displacements of local coordinate systems are given to the absolute coordinate system O_Ax_Ay_Az_A. For systems of Fig. 1 and Fig. 2 make the following assumptions [11]:

- elements of the system are solids;

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anti-roll bars are massless and their stiffness is regarded as equivalent spring connected to the arms at point to a distance $L_{sf}$ of the joint (hinge) of the front axle and $L_{sb}$ of the joint of the rear axle;

- give an account damping and elastic properties of the main elements $c_{rf}$, $c_{rb}$, $\beta_{rf}$, $\beta_{rb}$, respectively, springs and shock absorbers the front and the rear axle, and the elasticity of the tire $c_{gf}$ and $c_{gb}$ the front and the rear axle;

- elastic and damping elements have linear characteristics;
• system is placed in a equilibrium position as the centers of gravity to the wheels lie on a horizontal axis. $O_1y_1$ axis coincides with the axis $O_2y_2$, and $O_3y_3$ axis coincides with $O_4y_4$.

For generalized coordinate systems are adopted:

• $z_0$ - linear displacement of the local coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ on axis $O_z$;

• $\varphi_0, \psi_0$ - angular displacement of the local coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ respectively around the axes $O_x$ and $O_y$;

• $\varphi_1$ - angular displacement around the axis $O_1x_1$ of the coordinate system $O_1x_1y_1z_1$;

• $\varphi_2$ - angular displacement around the axis $O_2x_2$ of the coordinate system $O_2x_2y_2z_2$;

• $\varphi_3$ - angular displacement around the axis $O_3x_3$ of the coordinate system $O_3x_3y_3z_3$;

• $\varphi_4$ - angular displacement around the axis $O_4x_4$ of the coordinate system $O_4x_4y_4z_4$;

• $\psi_1$ - angular displacement around the axis $O_1y_1$ of the coordinate system $O_1x_1y_1z_1$;

• $\psi_2$ - angular displacement around the axis $O_2y_2$ of the coordinate system $O_2x_2y_2z_2$;

• $\psi_3$ - angular displacement around the axis $O_3y_3$ of the coordinate system $O_3x_3y_3z_3$;

• $\psi_4$ - angular displacement around the axis $O_4y_4$ of the coordinate system $O_4x_4y_4z_4$;

To find laws of motion in the absolute coordinate system $O_Ax_Ay_Az_A$ is necessary to define the transition matrices of each local coordinate systems to the absolute.

Matrix of transition from $O_0x_0y_0z_0$ to $O_Ax_Ay_Az_A$ for Fig. 1 and 2 is:

$$ T_0^A = \begin{bmatrix} \cos \psi_0 & 0 & -\sin \psi_0 & 0 \\ -\sin \varphi_0 \sin \psi_0 & \cos \varphi_0 & -\sin \varphi_0 \cos \psi_0 & 0 \\ \cos \varphi_0 \sin \psi_0 & \sin \varphi_0 & \cos \varphi_0 \cos \psi_0 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(1)

$x_0$ and $y_0$ are zero because is consider only linear oscillation on axis $O_z$, i.e. only vertically;

Matrix of transition from $O_1x_1y_1z_1, O_2x_2y_2z_2, O_3x_3y_3z_3, O_4x_4y_4z_4$, to $O_0x_0y_0z_0$ have a type:

For Figure 1:

$$ T_1^0 = \begin{bmatrix} 1 & 0 & 0 & L_f \\ 0 & \cos \varphi_1 & -\sin \varphi_1 & -b_f \\ 0 & \sin \varphi_1 & \cos \varphi_1 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^0 = \begin{bmatrix} \cos \psi_3 & 0 & -\sin \psi_3 & -L_b \\ 0 & 1 & 0 & -b_b \\ \sin \psi_3 & 0 & \cos \psi_3 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(2)

$$ T_2^0 = \begin{bmatrix} 1 & 0 & 0 & L_f \\ 0 & \cos \varphi_2 & -\sin \varphi_2 & b_f \\ 0 & \sin \varphi_2 & \cos \varphi_2 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_4^0 = \begin{bmatrix} \cos \psi_4 & 0 & -\sin \psi_4 & -L_b \\ 0 & 1 & 0 & b_b \\ \sin \psi_4 & 0 & \cos \psi_4 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix} $$
For Figure 2 only $T^0_3$ and $T^0_4$ are different:

$$T^0_3 = \begin{bmatrix} 1 & 0 & 0 & -L_{b_0} \\ 0 & \cos \varphi_3 & -\sin \varphi_3 & -b_{b_0} \\ 0 & \sin \varphi_3 & \cos \varphi_3 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_4 = \begin{bmatrix} 1 & 0 & 0 & -L_{b_0} \\ 0 & \cos \varphi_4 & -\sin \varphi_4 & b_{b_0} \\ 0 & \sin \varphi_4 & \cos \varphi_4 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (3)

After multiplying the matrices and simplify the resulting expressions for the components of the angular velocity of arms to three axes:

For Figure 1:

- front right arm:

$$\omega^A_{1x} = \dot{\varphi}_0 + \dot{\varphi}_1 \cos \psi_0$$

$$\omega^A_{1y} = \dot{\psi}_0 \cos \varphi_0 + \dot{\varphi}_1 \sin \psi_0 \sin \varphi_0$$

$$\omega^A_{1z} = \dot{\varphi}_1 \sin \psi_0 \cos \varphi_0 - \dot{\psi}_0 \sin \varphi_0$$

After removal of terms of a higher order is received:

$$\omega^A_{1x} = \dot{\varphi}_0 + \dot{\varphi}_1$$

$$\omega^A_{1y} = \dot{\psi}_0$$

$$\omega^A_{1z} = 0$$  \hspace{1cm} (4)

Similarly to determine the angular velocities of the other arms:

- front left arm:

$$\omega^A_{2x} = \dot{\varphi}_0 + \dot{\varphi}_2$$

$$\omega^A_{2y} = \dot{\psi}_0$$

$$\omega^A_{2z} = 0$$  \hspace{1cm} (5)

- rear right arm:

$$\omega^A_{3x} = \dot{\varphi}_0$$

$$\omega^A_{3y} = \dot{\psi}_0 + \dot{\psi}_3$$

$$\omega^A_{3z} = 0$$  \hspace{1cm} (6)

- rear left arm:

$$\omega^A_{4x} = \dot{\varphi}_0$$

$$\omega^A_{4y} = \dot{\psi}_0 + \dot{\psi}_4$$

$$\omega^A_{4z} = 0$$  \hspace{1cm} (7)

For Figure 2 is only different angular speeds of the rear arms:

- rear right arm:
\[ \omega_{x}^{A} = \dot{\phi}_{0} + \dot{\phi}_{3} \quad \omega_{y}^{A} = \dot{\psi}_{0} \quad \omega_{z}^{A} = 0 \]  

(9)

- rear left arm:

\[ \omega_{x}^{A} = \dot{\phi}_{0} + \dot{\phi}_{3} \quad \omega_{y}^{A} = \dot{\psi}_{0} \quad \omega_{z}^{A} = 0 \]  

(10)

Kinetic energies of two systems are:

For Figure 1:

\[
T = \frac{1}{2} m_{0} \dot{z}_{0}^{2} + \frac{1}{2} J_{0x} \dot{\phi}_{0}^{2} + \frac{1}{2} J_{0y} \dot{\psi}_{0}^{2} + \frac{1}{2} J_{p} \dot{\psi}_{0}^{2} + \frac{1}{2} J_{p} \dot{\phi}_{3}^{2} + \frac{1}{2} J_{p} \dot{\phi}_{2}^{2} + \frac{1}{2} J_{p} \dot{\psi}_{3}^{2} + \frac{1}{2} J_{p} \dot{\psi}_{4}^{2} + \frac{1}{2} J_{p} \dot{\phi}_{1}^{2} + \frac{1}{2} \psi_{0}^{2} + \frac{1}{2} \psi_{3}^{2} + \frac{1}{2} \psi_{4}^{2} + \frac{1}{2} \phi_{0}^{2} + \frac{1}{2} \phi_{2}^{2}
\]

(11)

For Figure 2:

\[
T = \frac{1}{2} m_{0} \dot{z}_{0}^{2} + \frac{1}{2} J_{0x} \dot{\phi}_{0}^{2} + \frac{1}{2} J_{0y} \dot{\psi}_{0}^{2} + \frac{1}{2} J_{p} \dot{\psi}_{0}^{2} + \frac{1}{2} J_{p} \dot{\phi}_{3}^{2} + \frac{1}{2} J_{p} \dot{\phi}_{2}^{2} + \frac{1}{2} J_{p} \dot{\psi}_{3}^{2} + \frac{1}{2} J_{p} \dot{\psi}_{4}^{2} + \frac{1}{2} \psi_{0}^{2} + \frac{1}{2} \psi_{3}^{2} + \frac{1}{2} \psi_{4}^{2} + \frac{1}{2} \phi_{0}^{2} + \frac{1}{2} \phi_{2}^{2} + \frac{1}{2} \phi_{1}^{2} + \frac{1}{2} \phi_{1}^{2}
\]

(12)

Potential energies of the two systems are:

For Figure 1:
\[ \Pi = \frac{1}{2} c_{rf}(L_{cf} \phi_1)^2 + \frac{1}{2} c_{rf}(-L_{cf} \phi_2)^2 + \frac{1}{2} c_{rb}(L_{cb} \psi_3)^2 + \frac{1}{2} c_{rb}(L_{cb} \psi_4)^2 + \]
\[ + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_1 + b_{gf}) \psi_0 + L_f \psi_0 - b_{gf} \phi_1 - q_{f_1})^2 + \]
\[ + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_2 + b_{gf}) \psi_0 + L_f \psi_0 + b_{gf} \phi_2 - q_{f_2})^2 + \]
\[ + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_3 + b_{kb}) \psi_0 - (L_b + L_{kb}) \psi_0 - L_{kb} \psi_3 - q_{b_1})^2 + \]
\[ + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_4 + b_{kb}) \psi_0 - (L_b + L_{kb}) \psi_0 - L_{kb} \psi_4 - q_{b_2})^2 + \]
\[ + \frac{1}{2} c_{rf}(-L_{cf} \phi_1 - L_{cf} \phi_2)^2 + \frac{1}{2} c_{sb}(-L_{sb} \psi_3 + L_{sb} \psi_4)^2 \]

(13)

For Figure 2:

\[ \Pi = \frac{1}{2} c_{rf}(L_{cf} \phi_1)^2 + \frac{1}{2} c_{rf}(-L_{cf} \phi_2)^2 + \frac{1}{2} c_{rb}(L_{cb} \phi_3)^2 + \]
\[ + \frac{1}{2} c_{rb}(-L_{cb} \phi_4)^2 + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_1 + b_{gf}) \psi_0 + L_f \psi_0 - b_{gf} \phi_1 - q_{f_1})^2 + \]
\[ + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_2 + b_{gf}) \psi_0 + L_f \psi_0 + b_{gf} \phi_2 - q_{f_2})^2 + \]
\[ + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_3 + b_{kb}) \psi_0 - (L_b + L_{kb}) \psi_0 - b_{kb} \phi_3 - q_{b_1})^2 + \]
\[ + \frac{1}{2} c_{gz}(\nabla \cdot (b \phi_4 + b_{kb}) \psi_0 - (L_b + L_{kb}) \psi_0 + b_{kb} \phi_4 - q_{b_2})^2 + \]
\[ + \frac{1}{2} c_{rf}(-L_{cf} \phi_1 - L_{cf} \phi_2)^2 + \frac{1}{2} c_{sb}(-L_{sb} \phi_3 - L_{sb} \phi_4)^2 \]

(14)

The Rayleigh’s functions are:

For Figure 1:

\[ R = \frac{1}{2} \beta_{rf} (L_{cf} \phi_1)^2 + \frac{1}{2} \beta_{rf} (-L_{cf} \phi_2)^2 + \frac{1}{2} \beta_{rb} (L_{cb} \phi_3)^2 + \frac{1}{2} \beta_{rb} (L_{cb} \phi_4)^2 \]

(15)

For Figure 2:

\[ R = \frac{1}{2} \beta_{rf} (L_{cf} \phi_1)^2 + \frac{1}{2} \beta_{rf} (-L_{cf} \phi_2)^2 + \frac{1}{2} \beta_{rb} (L_{cb} \phi_3)^2 + \frac{1}{2} \beta_{rb} (-L_{cb} \phi_4)^2 \]

(16)

After applying Lagrange’s equation of 2nd kind:

\[ \frac{d}{dt} \left( \frac{\partial H}{\partial \dot{q}} \right) - \left( \frac{\partial H}{\partial q} \right) = - \left( \frac{\partial T}{\partial q} \right) - \left( \frac{\partial R}{\partial q} \right) \]

(17)
Comparative analysis on mathematical models...

For equations describing the laws of motion of the system is valid:

\[
\begin{bmatrix}
F_q \\
C_q
\end{bmatrix} = \begin{bmatrix}
\mathbf{J}_{ex} + m_p L_{mpf} & m_p L_{mpf} \\
-m_p L_{mpf} & \mathbf{J}_{ex} + m_p L_{mpf}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{J}_{ex} + m_p L_{mpf} & m_p L_{mpf} \\
-m_p L_{mpf} & \mathbf{J}_{ex} + m_p L_{mpf}
\end{bmatrix} \begin{bmatrix}
f_q \\
\dot{c}_q
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
f_q \\
\dot{c}_q
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
[m] = \begin{bmatrix}
\mathbf{J}_{ex} + m_p L_{mpf} & m_p L_{mpf} \\
-m_p L_{mpf} & \mathbf{J}_{ex} + m_p L_{mpf}
\end{bmatrix}
\]

\[
[m] = \begin{bmatrix}
\mathbf{J}_{ex} + m_p L_{mpf} & m_p L_{mpf} \\
-m_p L_{mpf} & \mathbf{J}_{ex} + m_p L_{mpf}
\end{bmatrix}
\]
- [C] is the matrix of elasticity, which is also symmetric and has dimension $7 \times 7$.

Cells colored in Lt Dwn Diagonal refer to the system of Figure 1 and those in gray (light) of Figure 2.

- [B] is the matrix of dissipative forces, showing the influence of damper - symmetric with dimension $7 \times 7$:  

```latex
\begin{array}{cccccc}
\ldots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\text{C}_{a1a1} & \text{C}_{a1b1} & \text{C}_{a1c1} & \text{C}_{a1d1} & \text{C}_{a1e1} & \text{C}_{a1f1} \\
\text{C}_{a2a1} & \text{C}_{a2b1} & \text{C}_{a2c1} & \text{C}_{a2d1} & \text{C}_{a2e1} & \text{C}_{a2f1} \\
\text{C}_{a3a1} & \text{C}_{a3b1} & \text{C}_{a3c1} & \text{C}_{a3d1} & \text{C}_{a3e1} & \text{C}_{a3f1} \\
\text{C}_{a4a1} & \text{C}_{a4b1} & \text{C}_{a4c1} & \text{C}_{a4d1} & \text{C}_{a4e1} & \text{C}_{a4f1} \\
\text{C}_{a5a1} & \text{C}_{a5b1} & \text{C}_{a5c1} & \text{C}_{a5d1} & \text{C}_{a5e1} & \text{C}_{a5f1} \\
\text{C}_{a6a1} & \text{C}_{a6b1} & \text{C}_{a6c1} & \text{C}_{a6d1} & \text{C}_{a6e1} & \text{C}_{a6f1} \\
\text{C}_{a7a1} & \text{C}_{a7b1} & \text{C}_{a7c1} & \text{C}_{a7d1} & \text{C}_{a7e1} & \text{C}_{a7f1} \\
\end{array}
```
To obtain natural frequencies of its system equations are presented in Cauchy normal form:

\[ \begin{align*}
\beta_{rf}L_{cf}^2 & = 0 \\
\beta_{rb}L_{cb}^2 & = 0
\end{align*} \]

To obtain natural frequencies of its system equations are presented in Cauchy normal form:

\[ y + Ly = 0 \] \hspace{1cm} (19)

Where \( L \) is:

\[ L = \begin{bmatrix}
M^{-1}B & M^{-1}C \\
I & O
\end{bmatrix} \] \hspace{1cm} (20)

The output parameters of the system are vibration displacement, vibration velocity, vibration acceleration and they are obtained from the equations:

\[ y + Ly = Y \] \hspace{1cm} (21)

Where \( Y \) is:

\[ Y = \begin{bmatrix}
M^{-1}F(t) \\
O
\end{bmatrix} \] \hspace{1cm} (22)

After integration of the system using the method of Runge-Kutta receive all decisions in a given time interval.
NUMERICAL INVESTIGATIONS

The main parameters and their numerical values are shown in table 1:

*Table 1:*

<table>
<thead>
<tr>
<th>№</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Suspended masses</td>
<td>$m_0$</td>
<td>1400 kg</td>
</tr>
<tr>
<td>2</td>
<td>Nosuspended masses</td>
<td>$m_p$</td>
<td>30 kg</td>
</tr>
<tr>
<td>3</td>
<td>Moment of inertia of the sprung masses around longitudinal axis (x-axis)</td>
<td>$J_{0x}$</td>
<td>550 kg.m^2</td>
</tr>
<tr>
<td>4</td>
<td>Moment of inertia of the sprung masses around transverse axis (y-axis)</td>
<td>$J_{0y}$</td>
<td>2000 kg.m^2</td>
</tr>
<tr>
<td>5</td>
<td>Moment of inertia of the unsprung masses on the front axle around x-axis</td>
<td>$J_{pxf}$</td>
<td>5 kg.m^2</td>
</tr>
<tr>
<td>6</td>
<td>Moment of inertia of the unsprung masses on the rear axle around x-axis</td>
<td>$J_{pxb}$</td>
<td>2 kg.m^2</td>
</tr>
<tr>
<td>7</td>
<td>Moment of inertia of the unsprung masses on the front axle around y-axis</td>
<td>$J_{pyf}$</td>
<td>2 kg.m^2</td>
</tr>
<tr>
<td>8</td>
<td>Moment of inertia of the unsprung masses on the rear axle around y-axis</td>
<td>$J_{pyb}$</td>
<td>5 kg.m^2</td>
</tr>
<tr>
<td>9</td>
<td>Vertical co-ordinate of the center of gravity of the unsprung masses in relation to joint of the arms</td>
<td>H</td>
<td>0,4 m</td>
</tr>
<tr>
<td>10</td>
<td>Horizontal co-ordinate of the center of gravity of the unsprung masses in relation to joint of the front arms</td>
<td>b_f</td>
<td>0,4 m</td>
</tr>
<tr>
<td>11</td>
<td>Horizontal co-ordinate of the center of gravity of the unsprung masses in relation to joint of the rear arms</td>
<td>b_b</td>
<td>0,6 m</td>
</tr>
<tr>
<td>12</td>
<td>Distance from the center of gravity to the front axle</td>
<td>L_f</td>
<td>1,1 m</td>
</tr>
<tr>
<td>13</td>
<td>Distance from the center of gravity to the rear axle</td>
<td>L_b</td>
<td>1,5 m</td>
</tr>
<tr>
<td>14</td>
<td>Length of the front arm</td>
<td>b_{kf}</td>
<td>0,42 m</td>
</tr>
<tr>
<td>15</td>
<td>Length of the rear arm</td>
<td>L_{kb}</td>
<td>0,42 m</td>
</tr>
<tr>
<td>16</td>
<td>Distance from the contact point of the rear wheel to joint of the arm</td>
<td>b_{kb}</td>
<td>0,2 m</td>
</tr>
<tr>
<td>17</td>
<td>Distance from the center of gravity of the front(f) and the rear(b) arm to the respective joint</td>
<td>L_{mp}</td>
<td>0,4 m</td>
</tr>
<tr>
<td>18</td>
<td>Distance from fixing point of the front(f) and the rear(b) main elastic element to the respective joint</td>
<td>L_c</td>
<td>0,3 m</td>
</tr>
<tr>
<td>19</td>
<td>Distance from fixing point of the front(f) and the rear(b) anti-roll bar to the respective joint</td>
<td>L_s</td>
<td>0,28 m</td>
</tr>
<tr>
<td>20</td>
<td>Radius of the front(f) and the rear(b) wheels</td>
<td>R_k</td>
<td>0,26 m</td>
</tr>
<tr>
<td>21</td>
<td>Stiffness coefficient of the main elastic elements of the front axle</td>
<td>c_{rf}</td>
<td>25000 N/m</td>
</tr>
</tbody>
</table>
Comparative analysis on mathematical models...

<table>
<thead>
<tr>
<th>№</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.</td>
<td>Stiffness coefficient of the main elastic elements of the rear axle</td>
<td>(c_{rb})</td>
<td>25000 N/m</td>
</tr>
<tr>
<td>23.</td>
<td>Stiffness coefficient of the tyre</td>
<td>(c_{gz})</td>
<td>125000 N/m</td>
</tr>
<tr>
<td>24.</td>
<td>Stiffness coefficient of the anti-roll bars of the front(f) and the rear(b) axle</td>
<td>(c_s)</td>
<td>20000 N/m</td>
</tr>
<tr>
<td>25.</td>
<td>Damping coefficient of the front(f) and the rear(b) shock absorbers</td>
<td>(\beta_r)</td>
<td>1900 N.s/m</td>
</tr>
</tbody>
</table>

The parameters are not measured by authors and are taken from literary sources cited below.

Natural frequencies of the systems:

0.8099 Hz (for Fig.1) / 0.9839 Hz (for Fig. 2) - frequency of linear oscillations of suspended masses on z-axis;
1.6722 Hz / 1.7286 Hz - frequency of angular oscillation of the sprung masses around x-axis;
1.1740 Hz / 0.8367 Hz - frequency of angular oscillation of the sprung masses around y-axis;
7.8614 and 7.8506 Hz / 7.8593 and 7.8832 Hz - angular frequency of the front arms;
8.3490 and 8.3100 Hz / 8.2327 and 8.3100 Hz - angular frequency of the rear arms;

Disturbing actions in the system are sinusoidal and are attached in the center of the contact patch of the tire with the road. They have the following form:

\[
q = q_0 (1 - \cos(vt))
\] (23)

\(q_0 = 0.02\) m - height of the amplitude of roughness;
\(v\) - circular frequency of the disturbing action:

\[
v = \frac{2\pi V}{S}, \text{ rad/s}
\] (24)

The frequency of the disturbing action expressed in hertz:

\[
v = \frac{1}{2\pi} \frac{2\pi V}{S}, \text{ Hz}
\] (25)

\(V\) - velocity of the car, m / s;
\(S\) - wavelength, m.

As the maximum accelerations are important, the investigated of the behavior of individual
elements of the system was conducted at a frequency effects similar to their natural frequencies. The results obtained for some of the accelerations are shown in figures below:

**Figure 3**: Linear acceleration of sprung masses on z-axis respectively of the models in Figure 1 and 2

**Figure 4**: Angular acceleration of suspended masses on y-axis respectively of the models in Figure 1 and 2

**Figure 5**: Angular acceleration of the front left arm of Figure 1 and 2
CONCLUSION

The generalization of the matrix of both automotive suspension may be used to create automated software to set it computing part and thus to accelerate the work in choosing the type of suspension and its elastic parameters.

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