THEORETICAL AND EXPERIMENTAL STUDY OF THE MECHANISMS USED IN THE CONSTRUCTION OF PLANETARY TRANSMISSION OF HYBRID CARS

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INTRODUCTION

The necessity of protecting the environment by limiting and controlling the gas emissions that contributes considerably to the greenhouse effect, by limiting the excessive fuel consumption from the limited fossil fuel reserves needs a new orientation to new technologies for powering future vehicles.

A promising solution on medium term is that of hybrid propulsion. This idea is in the attention of the researchers from the largest firms that are producing automobiles and also of the research centers.

The hybrid drive systems used at automobiles are systems where the energy required for self-propulsion is provided at least by two sources based on different principles of generating energy. The general components of a hybrid drive system are:

- a transformer of irreversible energy,
- a stocking system of reversible energy,
- a reversible coupling system.

The coupling system has the role of ensuring the energy transformation between the driving wheels and the other two components. It can be made:

- through an irreversible connection between the energy transformer and the driving wheels that make possible the self propulsion of the vehicle;
- through a reversible connection between the battery and the wheels, that is used for the self propulsion of the vehicle or for the recovery of the breaking energy;
- through an irreversible connection between the energy transformer and the battery that is used for recharge when the vehicle stops.

The Japanese constructors were the first who introduced hybrid vehicles in production. The THS (Toyota Hybrid System), is a solution that joins both electric and heat engine traction system. Coupling both thermal and electrical power is made by a planetary mechanism.

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THE STUDY OF PLANETARY MECHANISMS USED IN POWER SOURCES CONNECTIONS

KINEMATIC SCHEME

Two planetary mechanisms that can be used in transmitting power to a vehicle wheels are presented in figure 1. The hybrid auto-propulsion of the automobiles is made with a heat engine (MT) and an electrical engine (ME) that is powered from a battery with an accumulator (BA) of high voltage.

The accumulator battery is charged even during the displacement of the automobile because of an electric generator (GE) driven by the heat engine. The power sources (the heat engine, the electric generator, the electrical engine) can be coupled by the planetary mechanism with two degrees of loose (fig. 1 with a double satellite with external gearing or with a simple satellite with internal gearing (fig. 1, b). The mechanism from figure 1 b, is used in the construction of hybrid transmission for the hybrid vehicles Toyota Prius and Lexus.

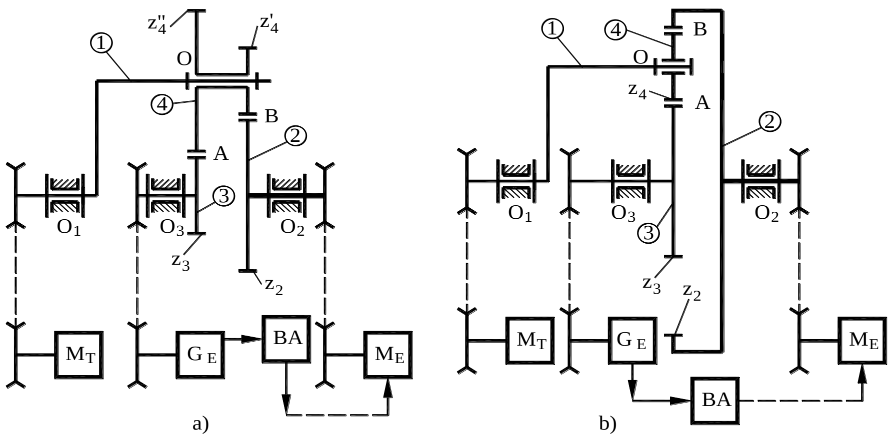

Figure 1: *Planetary mechanisms*

NOTATIONS

It is considered the coupling system from figure 1, *a* and the notations:

- J_i, $i = 1, 2, 3, 4$, axial moments of inertia of the elements marked with 1, 2, 3, 4;
- m_4, weight of the satellites 4;
- R_4, the length of the planetary carrier 1;
- z_2, z_3, z_4', z_4'' the number of teeth of the tooth wheels;
- i_1, i - the equations defined by the relations

$$i_1 = \frac{z_2}{z_4'}; i = \frac{z_2 z_4''}{z_3 z_4'} \tag{1}$$

- *A, B, C*, parameters - inertial parameters defined by the relations

$$\begin{aligned} A &= J_1 + m_4 R^2 + \left(1 + i_1^2\right) J_4 + (1 - i) J_3 \\ B &= -(1 - i) i J_3 + J_4 (1 + i_1) i_1 \\ C &= J_2 + J_3 i^2 + J_4 i_1^2 \end{aligned} \tag{2}$$

- $\theta_1, \theta_2, \theta_3$ the rotation angles of the elements 1, 2, 3,
- $\omega_1, \omega_2, \omega_3, \omega_4$ - the absolute angular velocities of the elements 1, 2, 3, 4,
- M_1, M_2, M_3 moments that act on the elements 1, 2, 3,

If the mechanisms 1, b are used then the modifications that are made are:

$$i_1 = -\frac{z_2}{z_4}; \; i = -\frac{z_2}{z_3} \tag{3}$$

and all the other notations remain the same.

THE KINETIC ENERGY AND THE GENERALIZED FORCES

Using the Wills method for the mechanism from figure 1 a, these relations are obtained

$$\frac{\omega_4 - \omega_1}{\omega_2 - \omega_1} = -\frac{z_2}{z_4'}; \; \frac{\omega_3 - \omega_1}{\omega_4 - \omega_1} = -\frac{z_4''}{z_3} \tag{4}$$

From which, with the notations (1) we deduct the equalities:

$$\begin{aligned} \omega_3 &= \omega_1 (1 - i) + \omega_2 i \\ \omega_4 &= \omega_1 (1 + i_1) + \omega_2 i_1 \end{aligned} \tag{5}$$

that are still available for the mechanism from figure 1, *b* is replaced with i_1 cu $-i_1$ *and* i cu $-i$. The kinetic energy of the system:

$$E_C = \frac{1}{2}(J_1\omega_1^2 + m_4R^2\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2 + J_4\omega_4^2) \quad (6)$$

with the notations (2) and (5) the equation becomes:

$$E_C = \frac{1}{2}\left(A\dot{\theta}_1^2 - 2B\dot{\theta}_1\dot{\theta}_2 + C\dot{\theta}_2^2\right) \quad (7)$$

The mechanic power at a certain time is given by:

$$P = M_1\omega_1 + M_2\omega_2 + M_3\omega_3 \quad (8)$$

or on the basis of the other relation (5)

$$P = \left[M_1 + M_3(1-i)\right]\omega_1 + (M_2 + M_3 i)\omega_2 \quad (9)$$

and from here we deduct the generalized equations:

$$Q_1 = M_1 + M_3(1-i); \quad Q_2 = M_2 + M_3 i. \quad (10)$$

DIFFERENTIAL EQUATIONS

Knowing the fact that:

$$\omega_1 = \dot{\theta}_1; \omega_2 = \dot{\theta}_2 \quad (11)$$

and using the Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_C}{\partial \dot{\theta}_i}\right) - \frac{\partial E_C}{\partial \theta_i} = Q_i,\ i = 1,\ 2 \quad (12)$$

we obtain the differential equations

$$A\dot{\omega}_1 - B\dot{\omega}_2 = M_1 + M_3(1-i); -B\dot{\omega}_1 + C\dot{\omega}_2 = M_2 + M_3 i \quad (13)$$

or

$$\dot{\omega}_1 = \frac{\lfloor M_1 + M_3(1-i)\rfloor C + (M_2 + M_3 i)B}{AC - B^2};\ \dot{\omega}_2 = \frac{\lfloor M_1 - M_3(1-i)\rfloor B + (M_2 + M_3 i)A}{AC - B^2} \quad (14)$$

THE MOVEMENT STUDY IN PERMANENT REGIME

The permanent movement is deducted from the conditions:

$$\omega_1 = 0\,;\omega_2 = 0 \quad (15)$$

which goes to this equations:

$$\begin{aligned}&\left[M_1 + M_3(1-i)\right]C + (M_2 + M_3 i)B = 0;\\ &\left[M_1 + M_3(1-i)\right]B + (M_2 + M_3 i)A = 0\end{aligned} \quad (16)$$

From the relations (16) is obtained the condition:

$$AC - B^2 > 0 \quad (17)$$

and then the equations (16) become:

$$M_1 + M_3(1-i) = 0\,; M_2 + M_3 i = 0 \quad (18)$$

The sources of power are present by the characteristics movement-angular speed through the relations $M_i = M_i(\omega_i)$ and goes to the values: ω_1^*, ω_2^*, ω_3^* of the angular velocities and there are obtained the permanent conditions, deducted from the equation system:

$$\begin{aligned}&M_1(\omega_1^*) + (1-i)M_3(\omega_3^*) = 0;\ M_2(\omega_2^*) + i\,M_3(\omega_3^*) = 0;\\ &\omega_3^* = (1-i)\omega_1^* + i\omega_2^*\end{aligned} \quad (19)$$

If these values $M_2(\omega_2^*)$, $M_3(\omega_3^*)$ are related to the value of the moment $M_1(\omega_1^*)$ of the heat engine, the following relations are obtained:

$$\frac{M_2(\omega_2^*)}{M_1(\omega_1^*)} = -\frac{i}{i-1}\,;\frac{M_3(\omega_3^*)}{M_1(\omega_1^*)} = -\frac{1}{i-1} \quad (20)$$

with the graphic representation from figure 2 and figure 3.

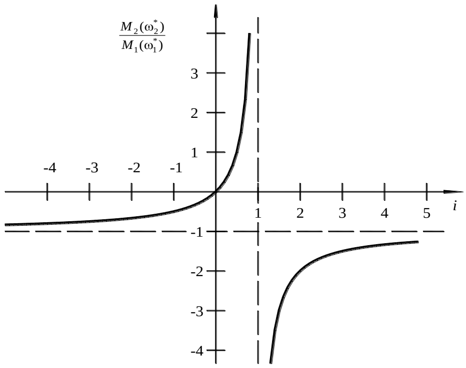

Figure 2: *Graphic* $\frac{M_2(\omega_2^*)}{M_1(\omega_1^*)} = -\frac{i}{i-1}$

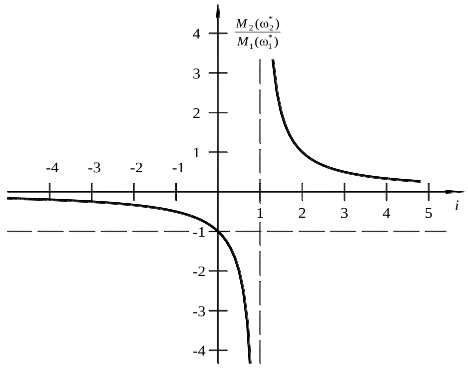

Figure 3: *Graphic* $\frac{M_3(\omega_3^*)}{M_1(\omega_1^*)} = -\frac{1}{i-1}$

The last relation (20) is represented in figure 4 for $i < 0$ and in figure 5 for $i > 1$.

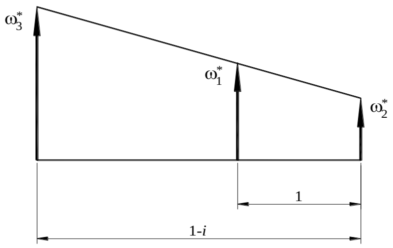

Figure 4**:** *Representation of the relation* $i < 0$

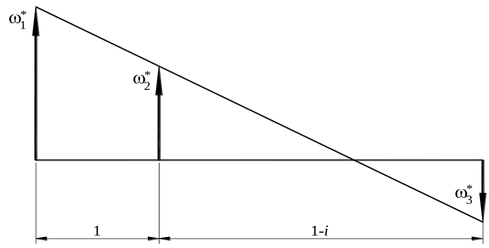

Figure 5**:** *Representation of the relation* $i > 1$

Starting from the supposition that the power of the engine $P = M_1\omega_1$ $P = M_1\omega_1$ is distributed to the generator and to the wheels sliding, it results that:

$$P_2 = M_2\omega_2 < 0\,;\, P_3 = M_3\omega_3 < 0 \quad (21)$$

$$P = |P_2| + |P_3| \quad (22)$$

and if it is admitted that $M_2 < 0$ then from figure 2 and 4 results that:

$$i < 0 \text{ or } i > 1 \quad (23)$$

If $i < 0$ results from figure 4 that $\omega_3^* > 0$ and from figure 2 that $M_3 < 0$ and because of that the value of P_2, P_3 is negative which means that the conditions (22) are respected.

If $i > 1$ from figure 4 results that $M_3 > 0$ and to respect the condition (22) is necessary that $\omega_3^* < 0$ (fig. 5) and:

$$\frac{\omega_2^*}{\omega_1^*} < \frac{i-1}{i} \tag{24}$$

Analogue is made for $i < 0$ and the complementary relation is obtained:

$$\frac{\omega_2^*}{\omega_1^*} > \frac{i-1}{i} \tag{25}$$

We can take into consideration at least two particular cases:

1. The heat engine works for charging the battery and the automobile is stopped $(\omega_2^* = 0)$.

Results ($i > 1$) and

$$\omega_3^* = -(i-1)\omega_1^* < 0 \tag{26}$$

and how $M_3(\omega_3^*) > 0$ (fig. 4) the systems functionality is normal, the power consumed by the generator is equal with the power given by the heat engine.

2. Starting the electrical engine first and then the heat engine move the automobile.

In the first faze $\omega_1 = 0$ and for $i > 1$ results that:

$$\omega_3^* = i\omega_2^* > 0 \tag{27}$$

Considering that the angular velocity to wheel, the wheel being operated by the electrical engine, has the same way as the angular velocity of the generator, then the moment M_3 at the generator is negative. Next, by starting the heat engine, we reached to the situation from figure 5 where $\omega_3 < 0$ and $M_3 > 0$.

THE MOVEMENTS STABILITY

The solution $\overset{*}{\omega}_1$, $\overset{*}{\omega}_2$, $\overset{*}{\omega}_3$ is obtained from the (19) system and is stabile even if $\Omega_i = \omega_i 0 - \overset{*}{\omega}_i$, $i = 1, 2, 3$ are going through zero. In this way it will be studied the stability after the first approximation. There are obtained the linear approximations:

$$\begin{aligned} M_1(\overset{*}{\omega}_1 + \Omega_1) &= M_1(\overset{*}{\omega}_1) + \Omega_1 M_{1p}(\overset{*}{\omega}_1); \\ M_2(\overset{*}{\omega}_2 + \Omega_2) &= M_2(\overset{*}{\omega}_2) + \Omega_2 M_{2p}(\overset{*}{\omega}_2); \\ M_3(\overset{*}{\omega}_3 + \Omega_3) &= M_3(\overset{*}{\omega}_3) + \Omega_3 M_{3p}(\overset{*}{\omega}_3) \end{aligned} \tag{28}$$

where by M_{ip} were denoted the functions derivates $M_i(\overset{*}{\omega}_i)$.

By replacing the equations (14) and taking into account the notations:

$$\begin{aligned} \alpha &= \frac{1}{AC - B^2}\left[M_{1p}(\overset{*}{\omega}_1) + (1-i)^2 M_{3p}(\overset{*}{\omega}_3)\right]; \\ \beta &= \frac{i(1-i)}{AC - B^2} M_{3p}(\overset{*}{\omega}_3);\ \gamma = \frac{1}{AC - B^2}\left[M_{2p}(\overset{*}{\omega}_2) + i^2 M_{3p}(\overset{*}{\omega}_3)\right] \end{aligned} \tag{29}$$

it is obtained the linear system of differential equations:

$$\dot{\Omega}_1 = (\alpha C + \beta B)\Omega_1 + (\beta C + \gamma B)\Omega_2;\ \dot{\Omega}_2 = (\alpha B + \beta A)\Omega_1 + (\beta B + \gamma A)\Omega_2 \tag{30}$$

The characteristic equation becomes:

$$\begin{vmatrix} \alpha C + \beta B - r & \beta C + \gamma B \\ \alpha B + \beta A & \beta B + \gamma A - r \end{vmatrix} = 0 \tag{31}$$

or

$$r^2 + Dr + E = 0 \tag{32}$$

where

$$D = -(\alpha C + 2\beta B + \gamma A);\ E = (\alpha C + \beta B)(\beta B + \gamma A) - (\alpha B + \beta A)(\beta C + \gamma B) \tag{33}$$

By making the calculations, the following results are obtained

$$D = -\frac{1}{AC - B^2}\left\{CM_{1p}(\omega_1^*) + AM_{2p}(\omega_2^*) + \left[i^2 A + 2i(1-i)B + (1-i)^2 C\right]M_{3p}\right\}$$
$$E = \frac{1}{AC - B^2}\left[M_{1p}(\omega_1^*)M_{2p}(\omega_2^*) + (1-i)^2 M_{2p}(\omega_2^*)M_{2p}(\omega_2^*) + i^2 M_{1p}M_{3p}\right] \quad (34)$$

For making the signs of the parameters *D*, *E*, there are taken into account the inequalities:

$$A > 0;\ C > 0;\ AC - B^2 > 0;$$
$$i^2 A + 2i(1-i)B + (1-i)^2 C > 0 \quad (35)$$

that are deducted from the element calculations from relation (2).

So, if $M_{ip}(\omega_i^*)$, $i = 1, 2, 3$ are negative, then $D > 0$; $E > 0$, the equation (32) has real negative solutions or complex solutions with the real part negative and then the movement is a stabile one.

The movement is unstable in the cases:

$$D > 0;\ E < 0, \qquad D < 0;\ E > 0, \qquad D < 0;\ E < 0 \quad (36)$$

THE CONSTRUCTIVE SOLUTION OF THE PROTOTYPE

The prototype has a mechanism constituted of a planetary mechanism with a double satellite (fig. 1, a) for coupling one heat engine and two electrical cars: a phase induction motor and an alternator.

The powers of the three engines is a result of a self-propulsion calculation of a vehicle with the performances: weight 600 kg, useful mass 200 kg, maximum speed 110 km/h, aerodynamic coefficient 0,3, rolling resistance 0,015 and frontal aria de 2,56 m^2. So the calculus resulted:

- the power of the heat engine: 15,3 kW la 3600 rot/min,
- the power of the electric engine: 7,5 kW la 3000 rot/min,
- the power of the electric generator: 6 kW la 8000 rot/min.

For obtaining compact mechanical solutions we have used a heat engine fully equipped, air cooled and a three-phased engine with outputs on both sides like in figure 6.

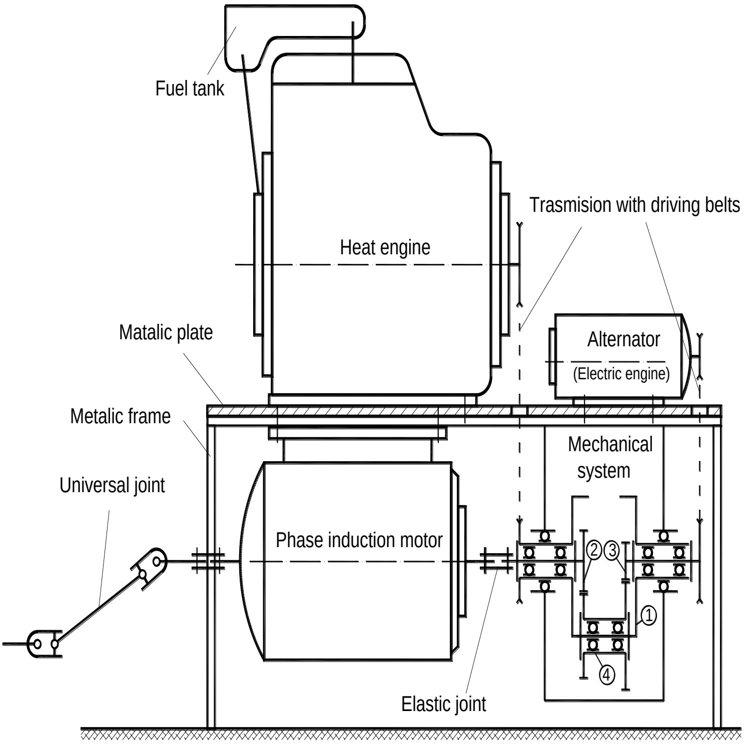

Figure 6: *The constructive scheme of the prototype*

In the same way the heat engine (MT from figure 1) will be located on the assembly: electric motor - mechanical system (ME – SM from figure 1).

The heat engine is coupled with the mechanical system using a transmission with driving belts, by separating the heat engine from the vibrations regarding the other parts of the mechanical system. Mechanically, the heat engine shaft is jointly in rotation with the port-satellite planetary arm mechanism.

The tooth wheels of the mechanical system have the following number of teeth: $z_3 = 20$ teeth, $z_2 = 35$ teeth, $z'_4 = 30$ teeth and $z''_4 = 45$ teeth. According to relation (1) the following values will result:

$$i = \frac{z_2 z''_4}{z_3 z'_4} = \frac{35 \cdot 45}{20 \cdot 30} = 2{,}625 \text{ and } i_1 = \frac{z_2}{z'_4} = 1{,}166$$

The electric generator (GE from figure 1) is a high power alternator or two alternators of medium power that are belt driven by the toothed wheel shaft joint (2). The generator is mounted in the upper part of the ensemble, on the same side with the heat engine.

The tensions of the two alternators are adjusted with two voltage relays, each alternator having its own accumulator battery. Next, the tensions of the two batteries are going to the consumers or to an inverter module that transforms alternative voltage into a three-phased direct current. There was never used such a module, three-phased voltage being available in the electric network of the test laboratory.

With the current configuration, where the generator is an alternator, an electric starter is necessary for the heat engine because the alternator isn’t electric reversible.

In the circuit of a battery the commanding system of the electric starter of the heat engine was also introduced.

The elements of the mechanism were designed and modeled in AutoCAD and in CATIA V5 environments, being also checked in terms of mechanical stresses in CATIA and LS-Dyna.

There were obtained numerical models and there was established a loading program in five phases.

AutoCAD modeling of all components of the mechanical system has a main advantage of simplifying the process of determining the mechanical sizes and the inertia of the components and ensembles. A second advantage is that of transferring the modules to the software specialized in the analysis of stresses and tensions.

Modeling the elements let to the determination of the mechanism constants, given by the relations (2):

$A = 0{,}00689117$, $B = 0{,}1110185$ and $C = 0{,}0760352$.

The differences appear between the sizes determined in AutoCAD and those experimentally obtained (weights, experimental determination of the inertia moments etc.) and are caused by not respecting the processing technology, the AutoCAD modeling being made with increased accuracy.

Modeling tooth wheels in AutoCAD is the synthesis of two scientific papers, the idea being inspired by the manufacture processes.

In figure 7 it is presented comparatively an image obtained with a camera and a photographic representation of the planetary gear mechanism.

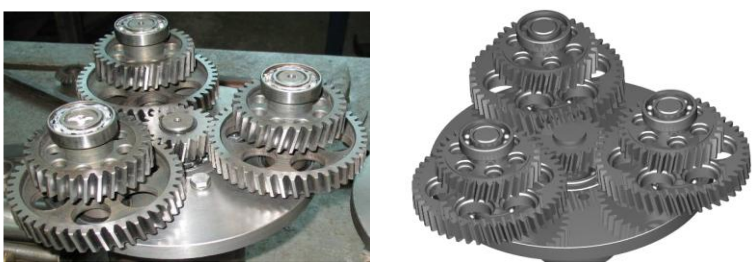

Figure 7: *The image obtained with a camera (left) and the photographic representation in AutoCAD of the gear mechanism (right).*

Mathematically speaking, this reflects real functioning cases of a vehicle with a hybrid propulsion system (heat engine, electric motor and electric generator).

The numerical results for the mechanism to function in transitory and permanent cases were obtained with a calculation program. This program is flexible, allowing not only to obtain the numerical results, but also to transfer the data to a program designed for graphical constructions.

For frequent functioning cases were compared the values experimentally obtained with those obtained using the mathematical model. The differences were lower the 5%.

The metallic frame with the mechanical system is presented in figure 8.

Figure 8: *The mechanical system on the frame*

Figure 9: *Speed setting module*

A second metallic frame (figure 9) was realized for mounting the batteries and the command module of the three-phased electric motor.

The power module SINAMICS allows the regulation of the asynchronous electric motor by modifying the voltage and varying the frequency. For a correct functioning the voltage data, maximum grip and power of the electric motor are inserted from the beginning in the CU240S command module. A particularity of the command module is that of retrieving the energy during the period when breaking the electric engine. The revolution can be both modified with a potentiometer or a computer by programming the functioning of the command module. Programming can be accomplished by setting switches, using the program and a computer directly plugged to the module or by an internet network.

EXPERIMENTAL DETERMINATIONS

The bench was coupled with two other existing benches. The coupling to the HOFFMAN bench with an electric break with tubular currents was realized by a universal joint and the coupling to the SCHENCK chassis dynamometer was realized by a homokinetic coupling with a tripod planetary.

The experimental determinations on the bench with an electric break with tubular currents had as main objective the analysis of the vibrations and sounds produced in operation.

From the frequency spectrograms we deduce the following:

For the housing of the mechanical system of coupling the power sources: the resonance box of the mechanical coupling system is at about 1000 Hz; in frequency spectrograms we found the frequencies caused by the gears engagement (revolution x number of teeth on the pinion x the number of satellites); the highest amplitude of a frequency is found at 2500 Hz.

For the frame of the bench: the heat engine induces harmonics on high frequencies; the structure of the frame has a side resonance of 2000Hz because of the constructive shape. This form is used only in this configuration, the frame being specially designed for the study of the two benches.

For the alternators: the vibrations are transmitted specially by mounting brackets; we have noticed some high frequencies because of the heat engine.

For the cylinder cap: the vibrations appear exclusively because of the excitation given by the heat engine; the values of the frequencies correspond to the resonance frequency of the cylinder cap.

For the fuel tank: in spectrograms we found a main resonance frequency of the fuel tank.

In terms of noise the following conclusions result:

In the case of bench functioning that it is driven by the heat and electric engine that are charged with nominal constant charge it is noticed: from the high frequencies spectrograms analysis results that there is noise due to engagement of the mechanical coupling system of the sources and ventilators of the electrical motors; predominates the low and medium frequencies; the global level isn’t high since the system is not body-worked as it will be in the functionally version on a vehicle or stationary ensemble.

In the case of studying transitional arrangements we find that: from the analysis of the global noise level results that there are small level modifications during the measures that are held in a steady regime; the heat engine increases the global noise level with less over 10 dB.

The housing vibrations of the mechanical system have been theoretically studied by using the equal sources method and by a dynamic analysis (in frequency) in Catia. The differences between the experimental results are low.

The experimental determinations on the chassis dynamometer had as main event measuring the cinematic and dynamic sizes of the mechanical system.

Being under construction (upgrading), in the hall in which the determinations were made the bench wasn't mounted below the ground level as in the project. There was made an ascension of the bench to make the coaxially between the shaft from the mechanical system and the fake deck with the rolling system.

The test report of the cinematic sizes has as main conclusion the functioning of the studied mechanism under the conditions of the firm's standards.

When the system is being used by the electric engine, the speed of the alternators is the speed of the electric motor multiplied with the gear ratio of 2.625. In this case the heat engine does not work and its speed is zero. When the system is being used by the heat engine, the speed of the electric motor and of the alternators is equal with that of the heat engine.

CONCLUSIONS

From those presented above it is noticed that planetary mechanisms with four elements can be used in joining thermo and electrical power sources. The advantages of these mechanisms are:

- a simple construction method,
- using this mechanism no longer needs the use of a gear box,
- there are not used breaks for blocking or unblocking some elements.

In terms of noise and vibrations, the mechanical system frames in the acceptable limits, the global noise being significantly lower then the one of a convention transmission.

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