THE STUDY OF THE SINGLE TRACK VEHICLES DYNAMICS

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INTRODUCTION

According to law of traffic safety of Serbia and related regulation on the classification of motor vehicles and trailer and technical condition for vehicles in traffic on the road, the single track vehicles are defined in article 6 and that: (1) type L_1 – moped with limited maximum velocity at 45 km/h, or working volume at 50 cm^3 for engine drive or limited maximum power at 4kW for electromotor drive, (2) type L_3 – motorcycle whose maximum velocity get over 45 kim/h, or working volume greater of 50 cm^3 for engine drive or maximum power greater of 4kW for electromotor drive. In article 14 above mentioned regulation are defined , (3) type K_2 – bicycle with human muscules power drive, (4) type K_3 – pedal drive vehicles with added electromotor drive whose maximum power is less than 0.25kW and maximum design velocity less than, which is not declared as vehicle type L_1.

The bicycles and motorcycles are the most represented categories of single track vehicles in road traffic whose dynamical characteristics in interactions with rider significant influence on the traffic safety. The study of the single track vehicles dynamics began on the 1800's and continues today. The first human – powerd single track vehicle cam in 1817, inveted by Baron Karl von Drais . The first pedal – driven two wheeler, in approximately 1840 is conected to name of the Scotsman Kirkpatrick Macmillan. The first motorcycle with a steam engine came in 1867 from American Sylvester Howard Roper and true motorcycle powered by means of an internal combustion engine is that in 1885, from Daimler and Maybach.

The motions of bicycles and motorcycles have many common attributes and are fundamentally different from other wheeled vehicles, as for example, double track vehicles. Handling and stability properties of rider – motorcycle system have been intense studied for more than 50 years. But there are many segments of these properties which are not fully investigated thus for, because of the system complexity. First of all, the dynamics of a motorcycle is very complicated themselves and difficult to study. Furthermore, the rider behaviour which include his motion with vehicle and relative to vehicle as and his control actions, affect the handling and stability of motorcycles. Though, considerable results are realized in this domain from 1970' s today.

The theoretical analysis of straight running stability for this system was conducted based on the Lagrangian approach to dynamic equation formulation by modeling the motorcycle as two rigid bodies[1]. The effects of any motorcycle parts rigidity to motorcycle lateral dynamics, incorporating structure compliance have been studied in [2], and rider body flexibility on the stability have been investigated in [3]. It was emphasized that when

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study stabilty, not only small disturbance forces from environment but also the input forces from the rider should be considered [4]. Motorcycle oscillatory behaviour is described during both running straight and cornering [5]. These cited papers shown that principal motions in motorcycle lateral directional dynamics is represented as results of four typical modes as follows: (1) capsize mode, involving a low frequency time constant which can be stable or unstable, (2) roll mode, involving a relatively fast time constant related to establishing and maintaining motorcycle roll angle, (3) weave mode, a second order motion involving combined roll and yaw motion, (4) wobble mode, a high frequency second order motion of the front fork subsystem about its rotation axis.

The rider control actions have also been studied by several authors [6], [7],[8] , and many importand facts were found. But actual riders control of the motorcycles is very complicated and influenced to traffic safety so this researching task has today priority, [10], [13].

1. GOAL OF INVESTIGATION

The goal of investigation in this paper is a study, according to above mentioned problems, of single track vehicle dynamics, especially of motorcycle dynamics. First of all, on the appropriate models conducted analysis of the rider – motocycle system, where rider has a passive role in sense to realize needful overall mass, as well as to introduce an input disturbance signal to stabilty study. Further, application of spectral theory to identification of motorcycle transfer function as system with multi input – multi output, what can contribute to: 1/ better understanding some dynamical phenomenon, [9], [11] 2/ design optimal multivariable controller, [12] 3/ forming alternative criterion to assessment motorcycle dynamics. Corresponding physical , simulation and identification models and used methodology to solving above mentioned research tasks are presented in next chapters.

2. MODELLING OF RIDER – MOTORCYCLE DYNAMICS

For description of motorcycle dynamics, physical model was developed as presented in Fig. 1, 2, 3. The system consists of the following concentrating masses: m_{11} – front wheel, m_{12} – complete front assembly including front fork, m_{21} – rear wheel, m_{22} – complete rear frame including drive assembly, m_{ru} – rider upper body, m_{rl} – rider lower body, m – mass of overall system . It is assumed that rider control of motorcycle motion by means of front fork steer torque and/or steer angle and upper body lean angle.

The origin position of the inertial reference coordinate frame, point 0, and axises direction are presented in Fig. 1. With motion variables specification, x, y, z, - longitudinal, lateral, vertical vehicle translation, respectively, ψ, η, ε, - corresponding vector of vehicle rotation, so called, roll, pitch, , yaw, respectively. In centre gravity c.g. of denoted masses in Fig. 1, are placed corresponding local coordinate frames as shown in Fig. 2, an example of system roll dynamics with focused rider upper mass, $m_{ru,}$ and motorcycle rear frame mass including of rider lower body mass, m_{22}^*. Fig. 3, shows the system motion relation in horisontal plane, with pointed out cornering phenomenon and side slip angles, δ_f, δ_r, α. The vector connections of the origin 0, with origins of the local coordinate frames, indicate to need coordionate transformation aimed to derivation of the differential equations. The dynamical system in Fig. 1, has seven degrees of fredoom denoted as, x, y, z, ψ, η, ε, β. With assumption $v_0 = x' = const$, and by free mode control, the number of degree –of- freedom degrees is reduced to four, treated as motorcycle lateral dynamics output variables , v, ψ, ε, β, and input variables, M, γ, in Fig. 4.

Besides in text given, in Fig. 1, 2, 3, 4, are employed following denotation, γ - rider lean angle, t_r – front wheel trail, h – high of c.g , χ - front fork angle in longitudinal plane, α - side slip angle at c. g, δ_1, δ_2 - front and rear wheel side slip angle, respectively. By reason of limited space, derivated differential equations in this paper are presented on the symbolic form, as implicite relation of the numerical parameters and coupled variables, suitable to fast check of simulation programme structure, as follows:

$$\dot{v} = f_1(p_v v, p_{\psi_2}\ddot{\psi}, p_\psi \psi, p_{\varepsilon_2}\ddot{\varepsilon}, p_{\varepsilon_1}\dot{\varepsilon}, p_{\beta_2}\ddot{\beta}, p_{\beta_1}\dot{\beta}, p_\beta \beta) \quad (1)$$

$$\ddot{\psi} = f_2(p_{v_1}\dot{v}, p_\psi \psi, p_{\varepsilon_2}\ddot{\varepsilon}, p_{\varepsilon_1}\dot{\varepsilon}, p_{\beta_2}\ddot{\beta}, p_{\beta_1}\dot{\beta}, p_\beta \beta, p_\gamma \gamma) \quad (2)$$

$$\ddot{\varepsilon} = f_3(p_{v_1}\dot{v}, p_v v, p_{\psi_2}\ddot{\psi}, p_{\psi_1}\dot{\psi}, p_\psi \psi, p_{\varepsilon_1}\dot{\varepsilon}, p_{\beta_2}\ddot{\beta}, p_{\beta_1}\dot{\beta}, p_\beta \beta) \quad (3)$$

$$\ddot{\beta} = f_4(p_{v_1}\dot{v}, p_v v, p_{\psi_2}\ddot{\psi}, p_{\psi_1}\dot{\psi}, p_\psi \psi, p_{\varepsilon_2}\ddot{\varepsilon}, p_{\varepsilon_1}\dot{\varepsilon}, p_{\beta_1}\dot{\beta}, p_\beta \beta, p_M M) \quad (4)$$

As can see from (1) to (4), these equations are mutually manyfold coupled what indicate on the complexity of considered system. Numerical values of the coefficient, p_{ji}, include motorcycle design parameters and rider control properties. The rider lean angle disturbance, γ, is introduced in equation number 2, with influental coefficient, p_{γ}, and rider steer fork torque disturbance, M, in equation number 4, with coefficient p_M.

According to equations from (1) to (4), a model of rider – motorcycle is formed, peresented in Fig. 4, as a multivariable system with three potential inputs, M – steer torque, γ - rider lean angle, v_0 – motorcycle longitudinal velocity, which control rider and four outputs, v- lateral velocity, ε - yaw angle, ψ – roll angle, β - steer fork angle, including theirs first and second order time derivatives.

Model, presented in Fig. 4, is a good basis to simulation and identification research tasks of rider – motorcycle dynamics, in general case of simultaneous rider action in all above presented loop, steer, lean, velocity change. In this paper simulation analysis was carried out at constant longitudinal velocity during one simulation step, for given input parameters. However, longitudinal velocity was varied from one to other simulation step aimed to determine its effects to system behaviour. In this manner are reduced time costs to system analysis and control action of rider is made easy. But proposed methodology, by means of system theory, modern simulation technics, and advanced softwar tools , made possible to solve actual problems in the most complicated form as defined with model structure in Fig.4.

In order to determine partial effects on the motorcycle dynamics, in first phase study, in context of overall investigation of rider – motorcycle stability, controllability, namely, handling quality, essential to traffic safety, was identified motorcycle dynamical characteristics separately, for steer torque input and for rider lean angle input.

Simulation and identification results, for above introduced assumtion are given and interpreted in next chapter both in time and frequency domain. According to requirements of system stability, time response to typical inputs are presented. As well as, frequency response as transfer relation from specified input to relevant outputs are analysed with respect to system controllability.

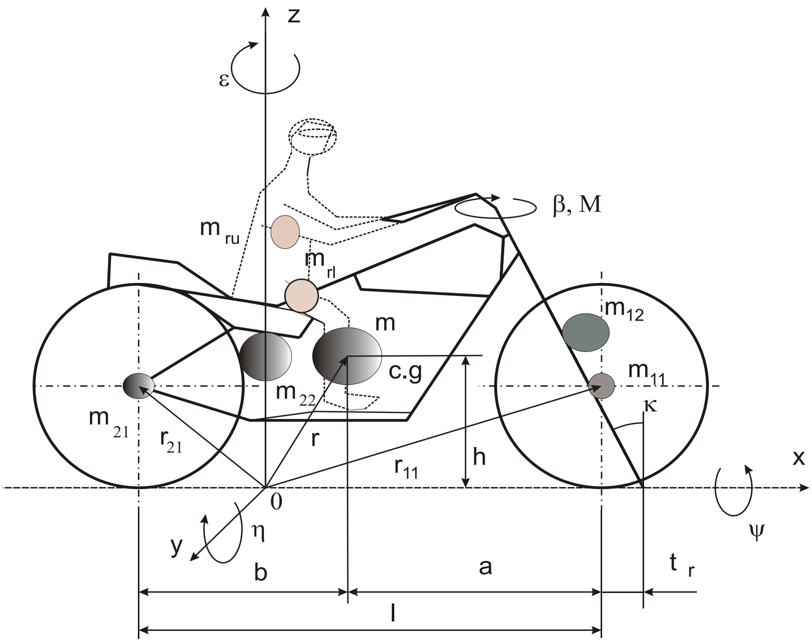


Figure 1: *Physical model of rider-motorcycle*

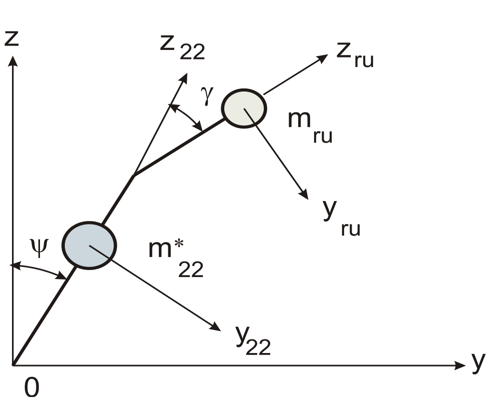


Figure 2: *Model of rider–motorcycle roll motion*

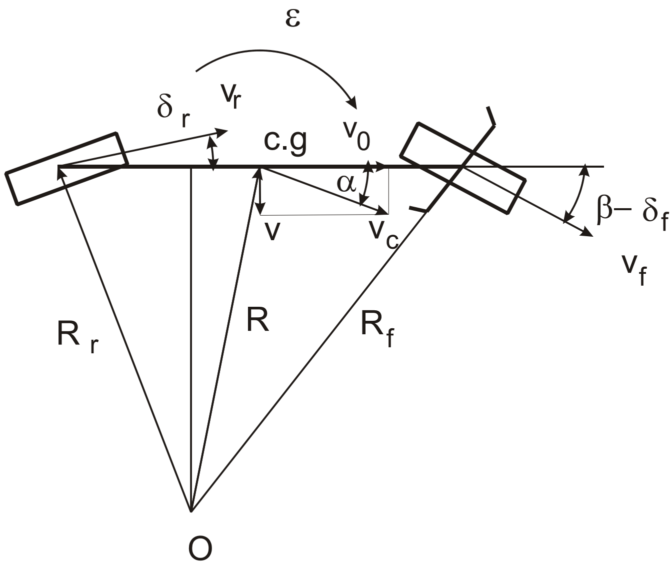


Figure 3: *Model of rider–motorcycle system motion in horisontal plane*

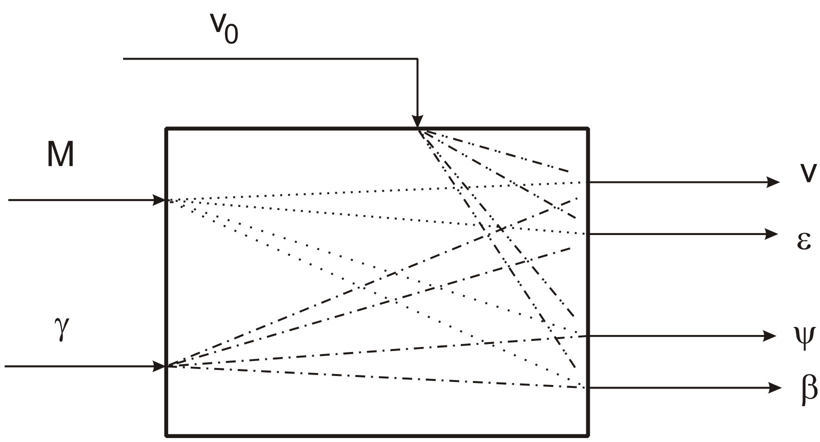


Figure 4: *Basic model to simulation and identification of rider–motorcycle system*

3. RESULTS

The structure and parameters of rider – motorcycle models, presented in Fig. 1, 2, 3, 4, made possible to study system performances for different combination input data. For example, different values of distribution and ratio of motorcycle – rider masses. Further, different geometric design parameters, tyre characteristics, driving speed, road and environment condition, as well as different from rider used control strategy.

As illustrative examples to verification proposed methodology, simulation research was conducted with input data of a mid class motorcycle at various forward speed. Presented results in this paper, on Figures from 5 to 26, are obtained at forward speed of 20 m/s. The results are grouped into three selected segments as follows: Fig. 5 – 16, time domain presentation, as system time response to typical input disturbance, Fig. 17 – 18, presentation by means of phase curve, Fig. 19 – 26, frequency domain presentation as frequency response of transfer function, from one input to chosen output. Typical system input disturbances were, in first case, an impulse steer torque on the motorcycle front fork, in second case, an impulse rider lean angle. The results for these two cases are compared and discussed.

Fig. 5 and 6, shown the change of lateral velocity versus time for two various input disturbances, (1) impulse steer torque, (2) impulse rider lean angle, respectively. There is essential difference between these two time response curves with respect to initial slope, number variations, asymmetry, segment shape at settling time and so on. Similar conclusions can be derived by comparing the changes of the rest curve pairs presented in time domain on the following Figures: 7, 8 – yaw rate, 9, 10 – roll rate, 11, 12 – steer rate. The slope of the flat curve segment at ending considered time interval idicates to system unstability, Fig. 5 – 10. These facts are more evident on the presentation of system position coordinate changes in Fig. 13, 14, as lateral displacement versus time, and Fig. 15, 16, as roll angle versus time, for two mode applied disturbance, respectively.

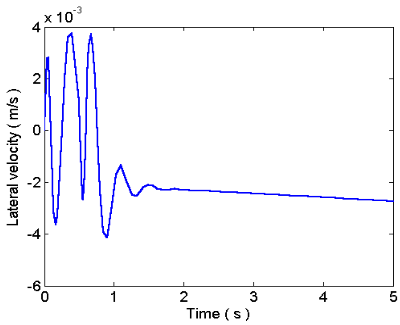


Figure 5: *Lateral velocity versus time Disturbance- impulse steer torque*

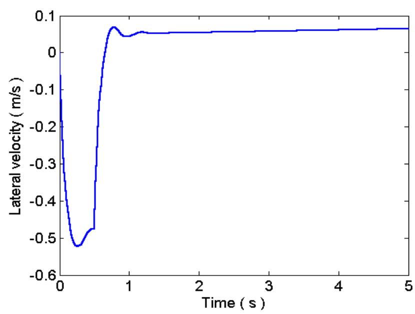


Figure 6: *Lateral velocity versus time. Disturbance- impulse rider lean angle*

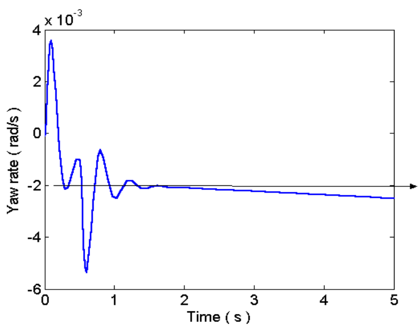


Figure 7: *Yaw rate versus time. Disturbance-impulse steer torque*

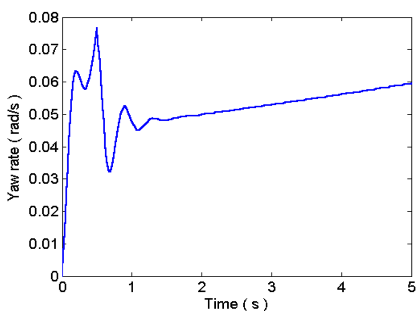


Figure 8: *Yaw rate versus time. Disturbance impulse rider lean angle*

According to presentation in Fig. 13, for first disturbance mode, initial lateral displacement values kept back limited during disturbace acting. After this moment lateral deviation from straightline direction increases rapidly. For other disturbance mode, as presented in Fig. 14, intesive changes of lateral deviation, during both transient stage, in opposite direction are evident.

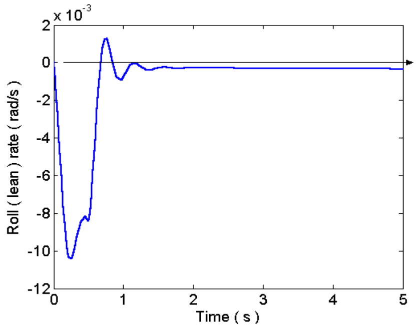


Figure 9: *Roll rate versus time. Disturbance-impulse steer torque*

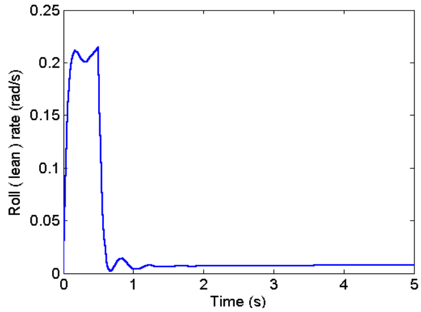


Figure 10: *Roll rate versus time. Disturbance-impulse rider lean angle*

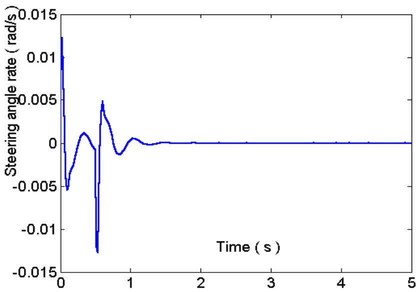


Figure 11: *Steer rate versus time. Disturbance-impulse steer torque*

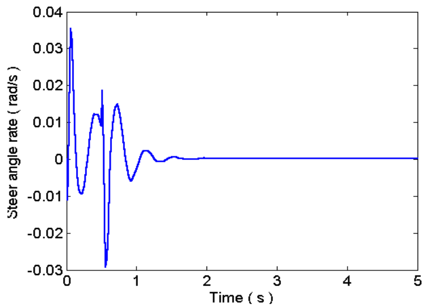


Figure 12: *Steer rate versus time. Disturbance-impulse rider lean angle*

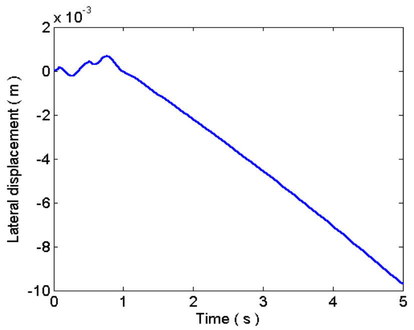


Figure 13: *Lateral displacement versus time. Disturbance-impulse steer torque*

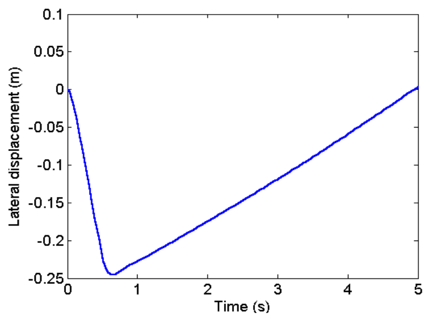


Figure 14: *Lateral displacement versus time. Disturbance-impulse rider lean angle*

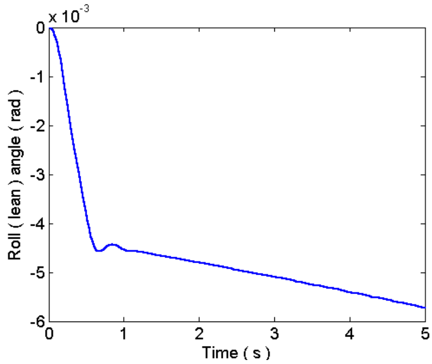
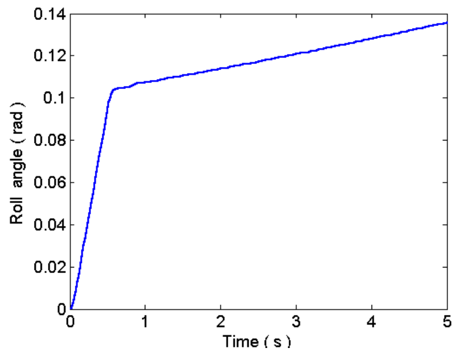


Figure 15: *Roll angle versus time Disturbance-impulse steer torque*



Figture 16: *Roll angle versus time. Disturbance-impulse rider lean angle.*

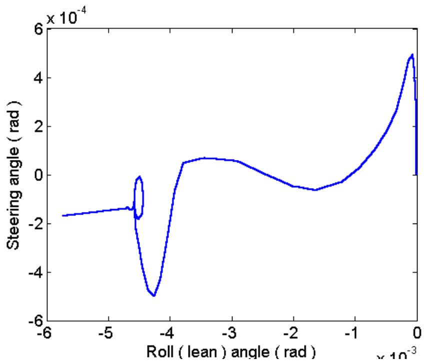


Figure 17: *Phase curve of roll angl–steer angle . Disturbance an impulse steer torque*

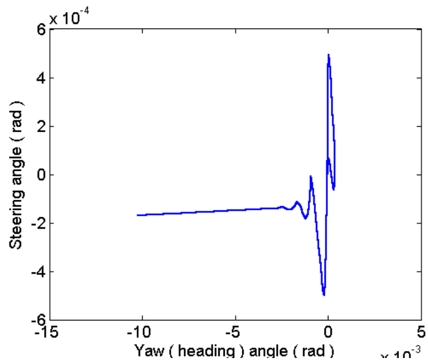


Figure 18: *Phase curve of yaw angle–steer angle. Disturbance an impulse steer torque.*

Fig. 15 and 16 shown the changes of roll angle versus time for two considered disturbance, above mentioned. The left curve consists two segments different slope according two typical transient intervals, placed in domain of negative roll angle values. The right curve consists also two segments different slope but is placed in domain of positive values of roll angle. The parameters of the curve changes in Fig. 13 – 16, are direct quantities of system unstability.

To assessment system stability can be also used the phase curves presented in Fig. 17 and 18, as relationship between, roll angle – steer angle and yaw angle – steer angle, respectively. As can see, after variation during first stage of transient process, steer fork angle converges to a steday state value, but roll angle, in Fig. 17 and yaw angle in Fig. 18, continue to increase. Namely, yaw angle diverges from straightline direction and roll angle, from upright system position.

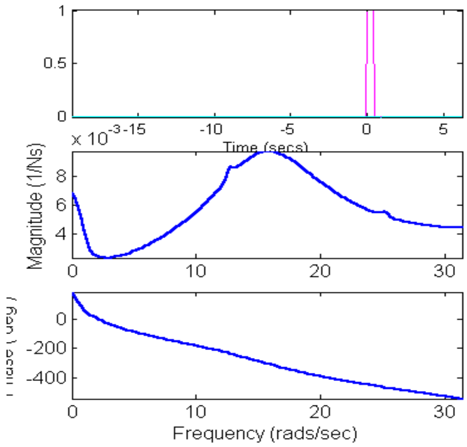


Figure 19: *Lateral velocity TF to steer torque input*

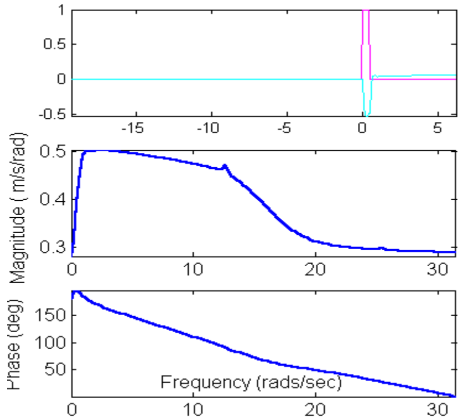


Figure 20: *Lateral velocity TF to rider lean angle input*

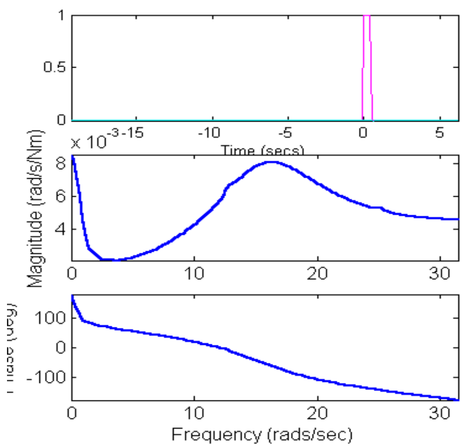


Figure 21: *Yaw rate TF to steer torque input*

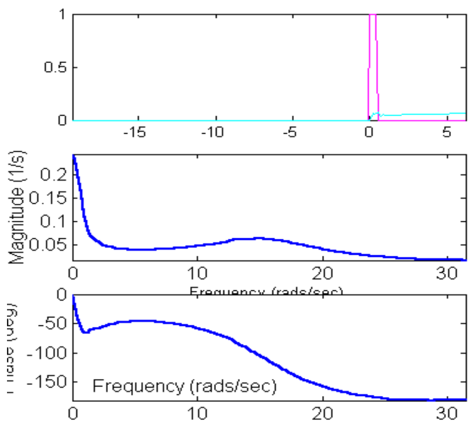


Figure 22: *Yaw rate TF to rider lean angle input*

The more information about system rider – motorcycle behaviour may be derived from results presented in frequency domain, in Fig. 19 – 26, as frequency response of transfer function of the observed output variable to steer torque – left curves and to rider lean angle – right curves.

The presented ilustrative samples of frequency response in Fig. 19 – 26, related to the relevant of system output variables as follows, lateral velocity, yaw rate, roll rate, steer rate, respectively, according to specification given in Fig. 4, for two introduced disturbances. In the same manner, as above pointed out for time domain, there is essential difference between system dynamical characteristics in rider seer an rider lean transfer channel.

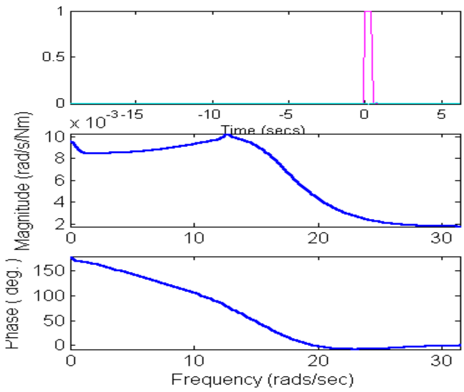


Figure 23: *Roll rate TF to steer torque*

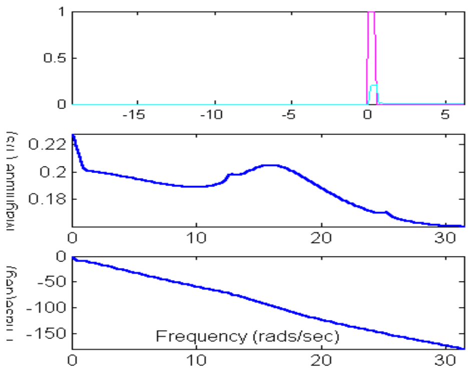


Figure 24: *Roll rate TF to rider lean angle*

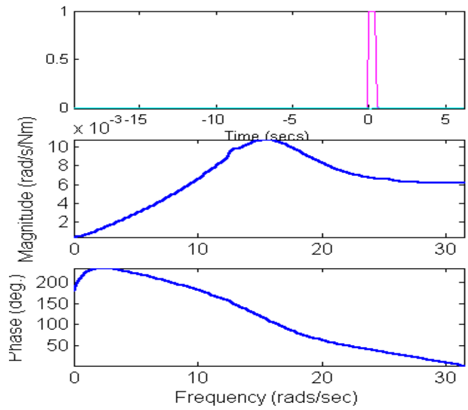


Figure 25: *Steer rate TF to steer torque input*

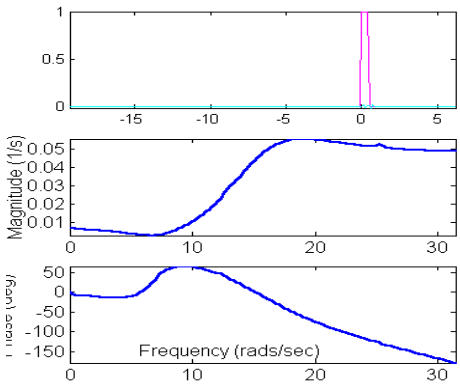


Figure26: *Steer rate TF to rider lean angle input*

The magnitude of system lateral velocity transfer function to steer torque input, as is shown in Fig. 19, possesses the properties of two connected filters, one with low narrow pass band characteristic, and other with mid pass band characteristic. Corresponding phase angle lead in the low pass band and lag in the mid and high pass band.

As shown in Fig. 20, magnitude of lateral velocty transfer function to rider lean angle as function of frequencies possesses the properties of the one broad pass band filter at low and mid frequencies, while corresponding phase angle lead in overall cosidered frequencies domain. On this basis, may be analyse the rest presentation in Fig. 21 – 26, with respect to rider – motorcycle handling quality and disturbance compensation possibilities.

4. CONCLUSIONS

The bicycles and motorcycles as the most represented categories of the sigle track vehicles in road traffic significant influence on the traffic safety. The dynamics a motorcycle is very comlicated themselves and difficult to study. The rider control behaviour more complicate the problems by analysis of motorcycle handling. By means of modern system theory, supported with advanced simulation technique and softwar tools, as shown in this paper, can be give important contribution to solve mentioned problems.

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