

# STATIC AND DYNAMIC ANALYSIS OF HYBRID METAL - COMPOSITE SHAFTS

Zorica Đorđević<sup>1</sup>, Nenad Kostić, Saša Jovanović, Vesna Marjanović, Mirko Blagojević

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## 1. INTRODUCTION

Composite materials are obtained by combining two or more materials, all in order to make new materials with controlled and more favourable characteristics. As a result, these materials have improved mainly mechanical properties, such as strength, stiffness, toughness, than their constituent parts. The greatest advantage of composite materials is reflected in the fact that most of them use the best features of the component materials and characteristics that themselves do not possess individually. Mainly, the formation of composite materials improve the following characteristics: strength, stiffness, corrosion resistance, abrasion resistance, weight reduction, lifetime, thermal isolation, acoustic isolation and increase elasticity modulus. Replacement of steel composite shaft is a novelty all over the world, especially in our country. Composite materials have been applied, as mentioned earlier, in aviation with increased accent on their usage analysis in the automotive industry and all other vehicle types. The above paper review is relating to some, the most characteristic, work in this field. Naveen Rastogi in his scientific research [1] presented a method for calculating the drive shaft, used in a car. He gave two important aspects of the input shaft of the budget: the budget and budget composite shaft coupling between the joints and back, discussing about solutions. Gubran H.B.H and Gupta K. [2] carried out theoretical and experimental studies on deflection and cross-section deformation of tubular composite shafts subjected to point static loading.

D.G. Lee together presented in [3] explained the new way of manufacturing a single piece of combined aluminium / composite drive shafts for cars with rear wheel drive. The composite material is composed of several layers that are made by the inner surface of the aluminium tubes. T. Rangaswamy and co-authors in the work [4] gave a realistic budget and analysis of composite drive shafts for power transmission, the optimal budget single piece drive shaft cars with rear-wheel drive using a highly modular and E-glass/epoxy (NM) carbon / epoxy composite. Gubran H.B.H. in [5] analyzed the deformation of the cross section of the shaft made of metal (steel or aluminium), composites (CFRP and GFRP) and combined return obtained from metals and composites. Special attention is devoted to the analysis of combined back. It is concluded that deformation of the shaft, and thus the deformation of cross sections, can be reduced if, instead of pure composite shaft, combined shaft is used (composite + metal).

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<sup>1</sup> Zorica Đorđević, University of Kragujevac, Faculty of Engineering, Sestre Janjić 6, 34000 Kragujevac, Serbia, email: zoricadj@kg.ac.rs

## 2. TYPES AND PROPERTIES OF COMPOSITES

Composites can be classified in different ways and by different criteria. From the point of this paper, the corresponding division of the composite would be deployed into the three main groups:

- composites with discrete particles embedded in a matrix,
- fiber-reinforced composites,
- layered composite laminates.

The characteristics of composites depend largely on three factors:

- strength and chemical stability of the matrix,
- strength and elasticity of reinforcing fibers,
- bond strength between the matrix and fibers arming.

Glass, carbon, organic (kevlar-aramid etc.) and boron fibers are the ones that are most often used for construction composites [4, 6, 7,...].

Glass fiber/polyester or glass fiber/epoxy resin composites are widely used in practice. The advantages of glass fibers over other materials are the following: easy and cheap production, possibility to produce very long fibers, high resistance to impact, etc. The basic disadvantages of glass-fiber-reinforced composites lie in the fact that they have a low modulus of elasticity and that they lose resistance at elevated temperatures.

The need for stronger materials led to greater application of carbon fiber/epoxy resin composites. Carbon fibers have high strength, high modulus of elasticity, low density, excellent mach inability, resistance to elevated temperatures, low thermal expansion coefficient, etc. Their main disadvantages are low toughness, high anisotropy, which causes additional problems to the constructor, and a high production cost compared to glass fibers.

The best-known organic fibers are aramid fibers, such as Kevlar. Aramid fibers have extremely high tensile strength, low density, excellent impact resistance, excellent isolating and heat properties, they are stable at a wide temperature range, they neither melt nor shrink, they are easy to machine and they can be produced in the form of weave. Their disadvantages are low compression, rather low modulus of elasticity, they are difficult to obtain, and they have a high production cost.

Boron fibers are characterised by high compression strength and torsion resistance. They have a positive coefficient of linear expansion. Their disadvantages are difficulty to machine and shape due to their high hardness, and a high production cost.

## 3. THE DESIGN OF THE COMPOSITE SHAFT

The composite shaft often has a shape of laminated cylindrical shell. It is important to derive the expressions for stress calculation and deformations in case of such laminate composite shaft. Those equations were derived by *Chang* and *Chen* [6] using the *Hamilton's* principle.

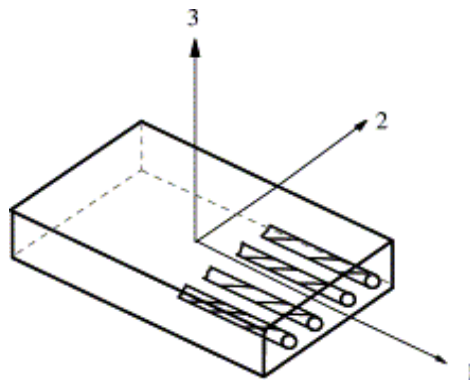
The constitutive relations for a lamina (see Figure 1) in the principal material directions (indicated by 1, 2 and 3) are given by:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} \quad (1)$$

The above equation will abbreviated as:

$$\{\sigma\} = [Q]\{\epsilon\} \quad (2)$$

where [Q] is the stiffness matrix.



**Figure 1:** A typical composite lamina and its principal material axes

Applying the Galerkin procedure the following matrix equations of motion of the spinning shaft system can be found:

$$[M]\{\ddot{q}\} + (\Omega[G] + [C])\{\dot{q}\} + [K]\{q\} = \{F\} \quad (3)$$

where [M] represents the mass matrix, [G] the gyroscopic matrix, [C] the damping matrix, [K] the stiffness matrix, {F} the external force vector and {q} the displacement vector.

The lamina is thin it is considered as the plane stress problem. Hence, it is possible to reduce the 3-D problem into 2-D problem. For unidirectional 2-D lamina, the stress-strain relationship in terms of physical material direction is given by:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad (4)$$

The matrix [Q] is referred as the reduced stiffness matrix for the layer and its terms are given by:

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}; \quad Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}; \quad Q_{66} = G_{12}$$

For an angle-ply lamina, where fibers are oriented at an angle with the positive x-axis (Longitudinal axis of shaft), the stress strain relationship is given by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (5)$$

#### 4. APPLICATION OF NUMERICAL METHOD OF STATIC ANALYSIS OF COMPOSITE SHAFTS

Differential equations that describe the state of stress and state of deformity, especially when it comes to complex systems, often cannot be solved by analytic methods. In these cases a solution can be reached by applying numerical methods. The main feature of the numerical method consists of the fact that the fundamental equations of Theory of elasticity are solved in a similar, numerical way. One of the most used numerical methods is the finite elements one. In the discretization of the continuum, one type of finite elements or combination of more types can be used. For hollow shaft analysis, as the case with composite shaft is, commonly used elements are in the shell form, that is, in the form of multilayer shells. For layered shells is typical that, by using suitable fiber orientation along with small thicknesses, it is possible to achieve very high stiffness of the shell structure. In formulating a finite element multi-layered shell calculation of all sizes is done in layers [7-10].

All numerical methods involve the usage of computers and appropriate software packages. Nowadays everywhere in the world as well as in our country, there are many software packages developed, such as NX Nastran, SAP, PAK, I-DEAS, SESAM, COSSMOS / M, PAL2, ANSYS ...

#### 5. DEVELOPMENT OF MODELS FOR THE NUMERICAL ANALYSIS OF COMPOSITE SHAFTS

Basic characteristics of metal materials (steel and aluminum) and composite materials, that are commonly used for making shafts (carbon fiber / epoxy resin, glass fiber / epoxy resin, aramid fiber / epoxy resin, boron fiber / epoxy resin) are given in Table 1 [7].

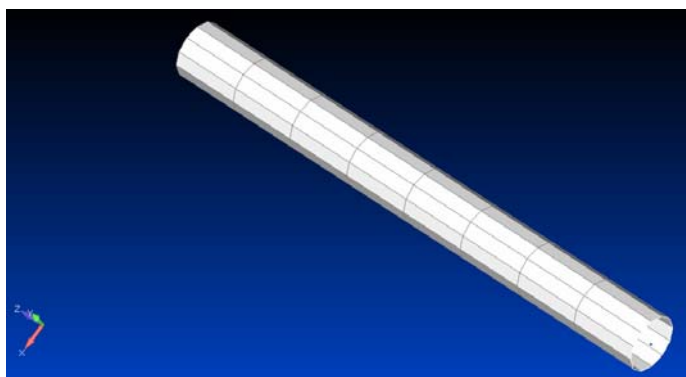
**Table 1:** Basic characteristic of material

Material	$E_1$ , MPa	$E_2$ , MPa	$G_{12}$ , MPa	$\nu$	$\rho$ , kg/m <sup>3</sup>
Steel	210000	210000	83000	0,3	7830
Aluminium	70000	70000	28000	0,28	2600
Carbon fiber / Epoxy resin	131600	8200	4500	0,281	1550

Glass fiber / Epoxy resin	43300	14700	4400	0,3	2100
Aramid fiber / Epoxy resin	81800	51000	1510	0,31	1380
Born fiber / Epoxy resin	211000	24100	6900	0,36	1967

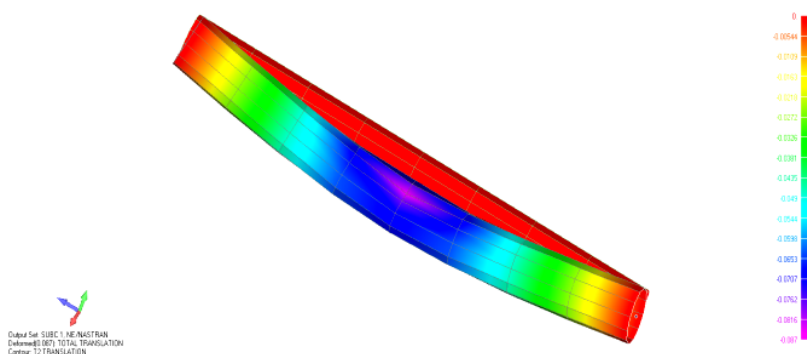
The labels in the table are:  $E_1$ -modulus in the longitudinal direction;  $E_2$  - modulus of elasticity in the transverse direction;  $G_{12}$ -modulus,  $\nu$  - Poisson's ratio,  $\rho$ -density of material. Dimensions of the shaft, analyzed in this paper are: length of the shaft is 1000 mm, mean radius is 50 mm, wall thickness of the ring cross section is 4 mm. The shaft is supported at the ends and the middle of the span is subject to static load of 1000N.

Model of analyzed composite shaft, obtained in the program package FEMAP v9.3 is shown in the Fig.2 [10]. The analysis used rectangular isoparametric final elements of multilayer shells form so that the shaft is divided into 8 elements in the axial and 12 elements in the circular direction.

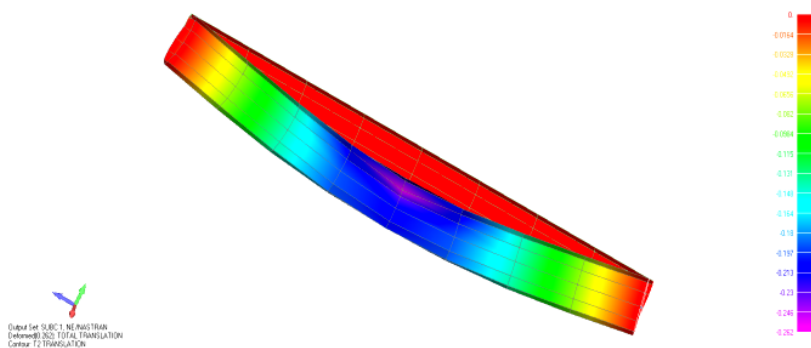


**Figure 2:** Composite shaft model

The process of analysis has began with metal shaft example (steel and aluminum). Figures 3 and 4 show the deformed shape and value displacement of these shaft materials.

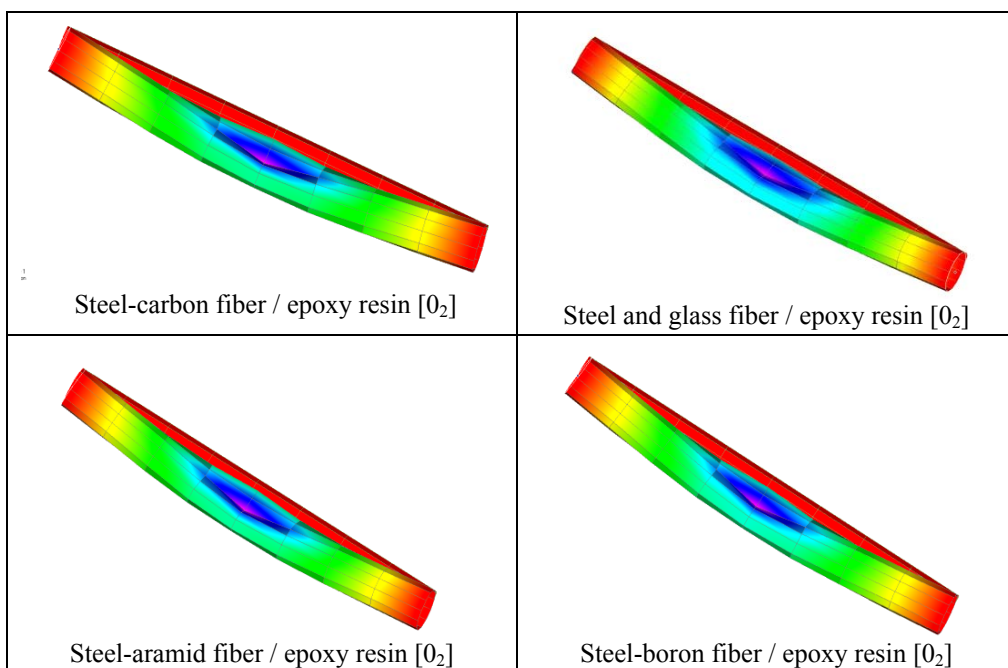


**Figure 3:** Value displacement of the steel shaft

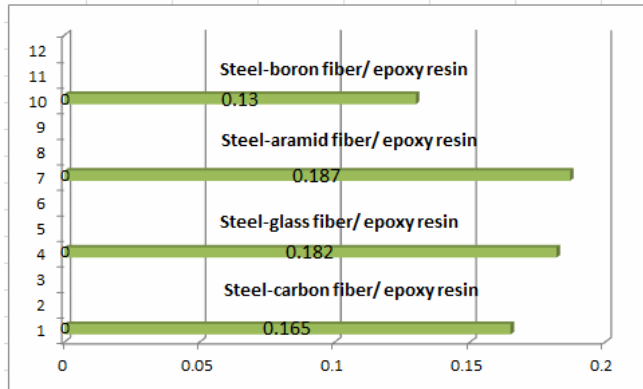


**Figure 4:** Value displacement of the aluminum shaft

According to figures 5 and 6, the appearance of deformed shafts and value displacement is given in case of steel and composite materials combination with fiber orientation by  $0^\circ$  and  $30^\circ$ . Results which are presented in figure 6 and figure 7 are created on the same way like for orientation fiber at  $0^\circ$ .

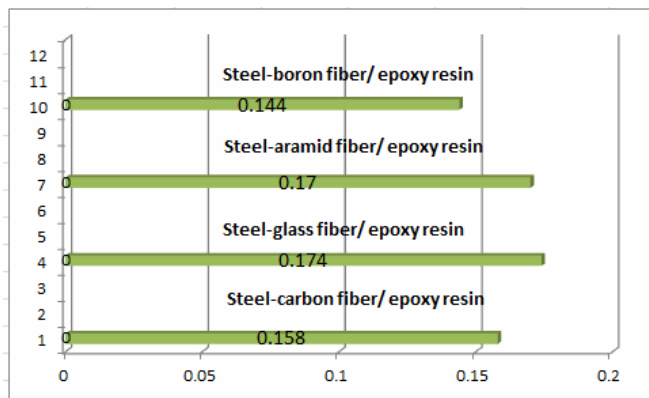


**Figure 5:** Value displacement of the shaft in the case of hybrid fiber orientation by  $0^\circ$



**Figure 6:** Value displacement of the shaft in the case of hybrid fiber orientation of 30°

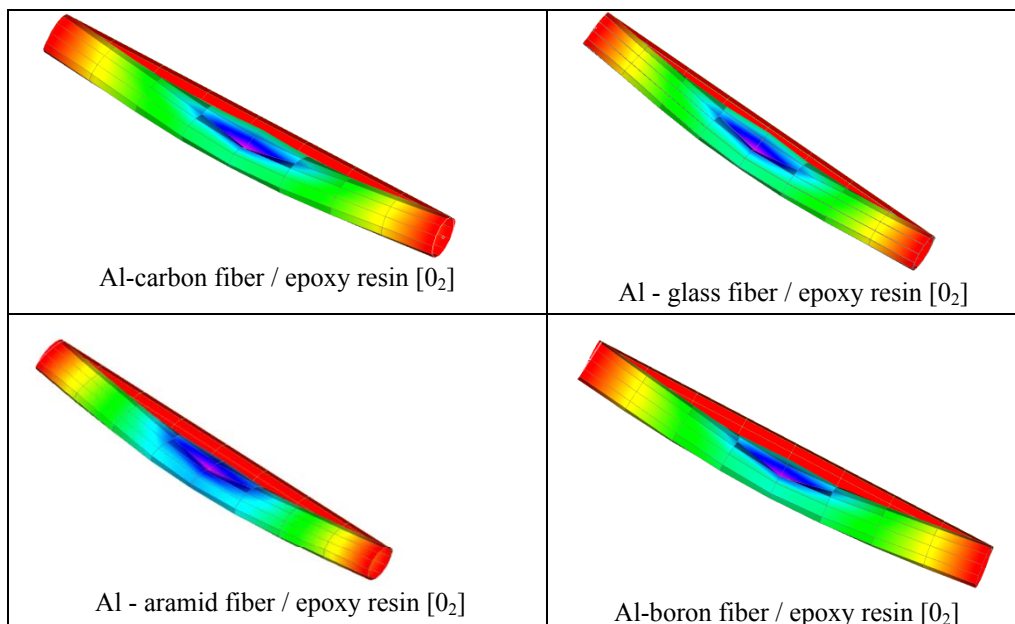
In the 90 degree fiber orientation, value displacement obtained for the case of hybrid shafts are shown in figure 7.



**Figure 7:** Value displacement in the case of hybrid fiber orientation by 90°

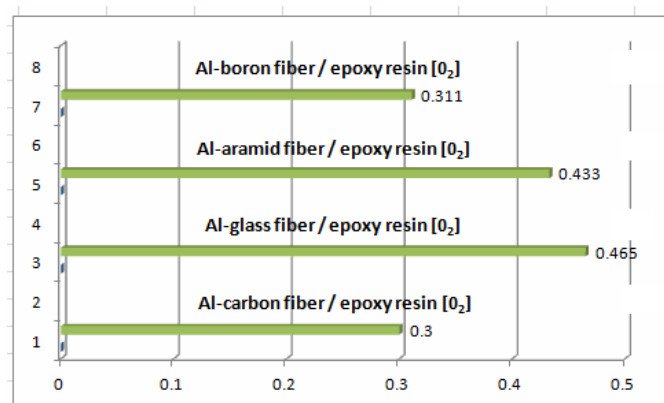
It can be concluded that in the case of the shaft obtained from a steel and composite materials combination, the best results, that is, the minimum deflection value boron fibers have, whereas, the worst still are aramid fiber / epoxy resin.

It can also be concluded that all have shafts have approximately similar displacement values combined with steel and composite materials with fiber orientation angles by 90°. This is explained by the fact that in the case of fiber orientation at 90°, elasticity modules in the transverse direction of the composite material is very small compared to steel. Therefore, in that case, displacement value, as well as shaft deformation depends a lot on the content of the steel part.



**Figure 8:** Displacement value of shaft in the case of hybrid fiber orientation at 0°

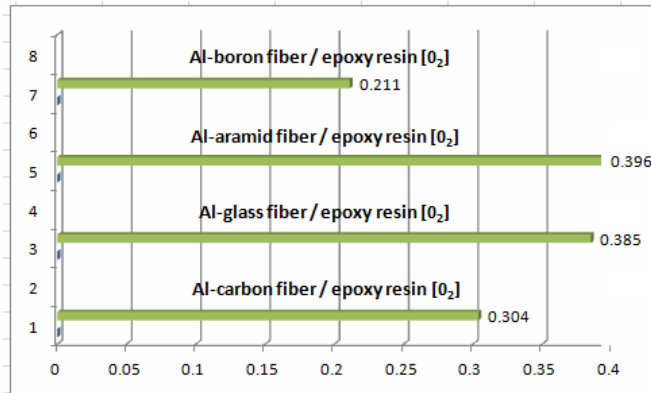
In figures 8 and 9, the appearance of deformed shaft and value displacement are given in case of aluminium and composite materials combination with fiber orientation at 0° and 30°. Results which are presented in figures 9 and 10 are created on the same way like for orientation fiber at 0°.



**Figure 9:** Displacement value of the shaft in the case of hybrid fiber orientation of 30°

In the fiber orientation at 90° degrees in case of combination of aluminum and composite materials obtained values of displacement are shown in Figure 10.





**Figure 10** : Displacement value of the shaft in the case of hybrid fiber orientation at 90°

The conclusions drawn for the case of combining steel and composite materials are also valid here. So, here is a recommendation to use a combination of aluminum and boron fiber orientation at angle of 0°. It should be noted that the density of aluminum is much lower than the density of steel, so this type of combined shaft is of much less weight and high durability, having for the result the combination more common in practice.

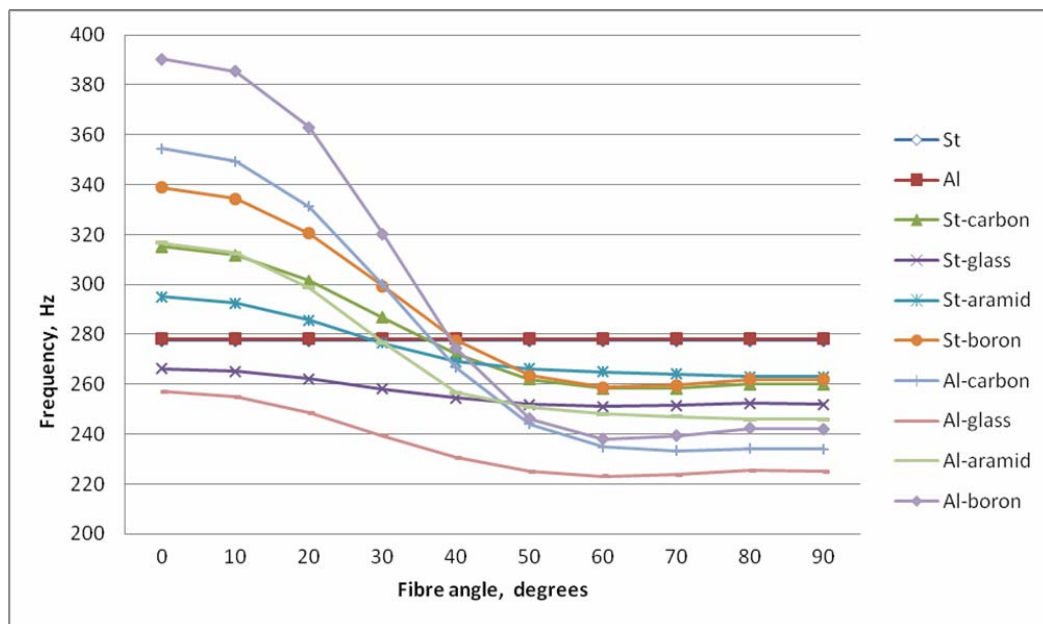
When it comes to laminate composites, as is this case here, it should be known that their strength depends on the strength of each layer which exists in it. Failure occurs in the layer with the most critical stress state. In order to eliminate the occurrence of failures in all laminate layers, it is necessary to define the permitted area for each layer. Resistance of laminated shell depends on the ratio of the initial failure FI; the smaller the fracture coefficient, the higher the resistance of the shell during the load impact. According to *Tsai-Wu* failure criterion, to avoid fracture in any layer, the condition is the FI < 1. This criterion is met for all types of analyzed materials and all the angles of fiber orientation.

## 6. FUNDAMENTAL NATURAL FREQUENCY

For the shaft, which has been subjected to static analysis, considered were also natural frequencies. It is known that the values of natural frequencies depend on the ratio  $E_1/\rho$  [5], [7]; the ratio is almost the same for the shaft made of steel or aluminum, hence the value of natural frequencies of the shaft of steel or aluminum are almost the same as it can be seen from Figure 11. However, in the case of the composites, ratio  $E_1/\rho$  varies, and depends on the orientation of the fibers, the maximum is for fibers angle of 0°, and decreases when the fiber orientation angle approaches 90°.

By analysis of Figure 11 can also be concluded that the worst characteristics, in terms of the value of natural frequencies, a hybrid composite shaft made of Steel-glass and Al-glass materials have. Given the small value of the  $E_1/\rho$  ratio, shaft natural frequencies of glass are for all values of fiber orientation angles less than metal ones.

Therefore, it can be concluded that the angle of fibers orientation has a major impact on the value of natural frequencies and the corresponding choice of the optimal fiber orientation angle can significantly affect the dynamic characteristics of the shaft [7], [11].



**Figure 11:** Natural frequencies of shafts made of different materials

## 7. CONCLUSIONS

The aim of this paper was to perform a complete analysis of the deformation state, as well as estimate of its own frequencies of composite shafts (made of carbon, glass, aramid and boron materials), and hybrid metal-composite shafts and compare their behavior with metal shafts (made of steel or aluminum).

Thanks to all the advantages that applying of composite material provide, like light weight, high strength and stiffness, good striking tenacity, better fatigue resistance, abrasion, vibration and acoustic resistance, it is so common nowadays for shafts to be made of composite material. It should be noted that the density of aluminum is much lower than the density of steel, so this type of combined shaft is of much less weight and high durability, having for the result the combination more common in practice.

## REFERENCES

- [1] Naveen Rastogi: „Design of composite drive shafts for automotive applications”, SAE World Congress Detroit, Michigan, 2004.
- [2] Gubran, H.B.H, Gupta, K.: „Cross-section deformation of tubular composite shafts subjected to static loading conditions”, Mechanics Research Communications, Vol. 29, No. 5, 2002, p. 367-374.
- [3] Lee D.G., Kim H.S., Kim J.W., Kim J.K.: „Design and manufacture of an automotive hybrid aluminum/composite drive shaft”, Composite Structures, Vol. 63, 2004.,p. 87-99.
- [4] Rangaswamy T., Vijayarangan S., Chandrashekar R.A., Venkatesh T.K., Anantharaman K.: „Optimal design and analysis of automotive composite drive shaft”,

- International Symposium of Research Students on Materials Science and Engineering, Chennai, 2002.
- [5] Gubran H.B.H.: „Dynamic of hybrid shafts”, *Mechanics Research Communications*, Vol 32, 2005., p. 368-374.
  - [6] Belingardi G., Obradovic J.: „Crash analysis of composite sacrificial structure for racing car”, *Journal Mobility & Vehicles Mechanics*, Vol. 37, No. 2, 2011, p. 41-55.
  - [7] Djordjevic, Z.: „Dynamic conduct composite shafts”, PhD Thesis, Faculty of Mechanical Engineering, Kragujevac, Serbia, 2008.
  - [8] Chang M.Y., Chen J.K., Chang C.Y.: „A simple spinning laminated composite shaft model”, *International Journal of Solids and Structures*, Vol. 41, 2004., 637-662.
  - [9] Milosavljevic D.: „Propagator matrix method in analisis of multilayered structures applicative in automotive engineering”, *Journal Mobility & Vehicles Mechanics*, Vol. 34, No.1- 2, 2008, p. 12-20.
  - [10] Kostic N.: „Static analysis of composite shaft”, Graduate thesis, Faculty of Mechanical Engineering, Kragujevac, Serbia, 2010.
  - [11] Lim J.W., Darlow M.S.: „Optimal sizing of composite power transmission shafting”, *Journal of American Helicopter Society*, 1986., 75-83.