CEPSTRUM ANALYSIS OF VIBRATION IN TRANSMISSION SYSTEM OF THE VEHICLE

Aleksandar Djurić, Snezana Jovanović[1]

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1. INTRODUCTION

1. 1 History of vibration

The origins of the theory of vibration can be traced back to the design and development of musical instruments (good vibration). It is known that drums, flutes, and stringed instruments existed in China and India for several millennia B.C. Also, ancient Egyptians and Greeks explored sound and vibration from both practical and analytical points of view.

The foundation of the modern day theory of vibration was probably laid by scientists and mathematician such as Robert Hooke, of the Hooke's law fame, who experimented on the vibration of strings. Sir Isaac Newton gave us calculus and the laws of motion for analysing vibrations. Daniel Bernoulli, Leonard Euler, Joseph Lagrange, Charles Coulomb, Joseph Fourier, Simeon-Dennis Poisson are some of the great scientists who work in vibration analysis. As a result of the industrial revolution and associated developments of steam turbines and other rotating machinery, an urgent need was felt for development in the analysis, design, measurement, and control of vibration. Motivation for many aspects of the existing techniques of vibration can be traced back to related activities since the industrial revolution. Much credit should go to scientists and engineers of more recent history, as well. Among the notable contributors are Rankin, Kirchhoff, Rayleigh, de Laval, Timoshenko, Crandall and others [1].

2. VEHICLE TESTING AND VIBRATION

Huge number of direct and indirect values has been measured and tracked in process of off-road vehicle prototype quality examinations methodology and testing. Analysis together with vehicle testing in real exploitation conditions has been improved, as measurement equipment quality has been increase. Highly sophisticated measurement technique is enabling direct and postprocess visualisation of all variations set up from load in extreme vehicle testing conditions.

Examination and testing has become an objective technical diagnose, where, as a part of support tools, has been import a vibration analysis test. This test represent diagnose of conditions by analyse of motion characteristics conditions of measured vibrations.

Method of vibration diagnosis enable re-establish correlation between dominant frequency (one or more) and mechanical defects which cause it. In that way, we can predict failure in vehicle transmission before it cause damage. In measurement and examination of vehicles that is very important, because it enable us to detect weak spots without dismantling transmission.

[1] *Aleksandar Djuric, MSc, Technical test center, Vojvode Stepe 445, 11000 Beograd, aleksandar_djrc@yahoo.com*

Vibrations, that being research, often are not forced like consequence of dynamic force influence, but combination of force and natural, resonant and modal vibrations. They are being generate by single source, so that recorded vibration signal at characteristic measurement position are heterodyning of several elementary vibrations with different frequency and intensity.

It is very hard to identify individual vibration generator in recorded time domain, so, thanks to absolute correlation of time and frequency region, recorded signals has been analysed in frequency domain through power spectrum that present in frequency axis decomposed energy. Practical merit of frequency spectrum is that enables detection and behaviour description of component part, which reveal existential large or slight problem.

In diagnostic matters vibrations defines: magnitude (expressed like shift, velocity or acceleration), frequency, phase and cast. Vibration level is function of shift and frequency. Vibration velocity is function of shift and frequency too, so it is gauge of vibration level and best indicator of part an/or condition of entire scheme. Velocity gives the most uniform spectrum, so we us it for vibration analysis and monitoring [2].

3. VIBRATION AND TRANSMISSION

3. 1 One degree of freedom model

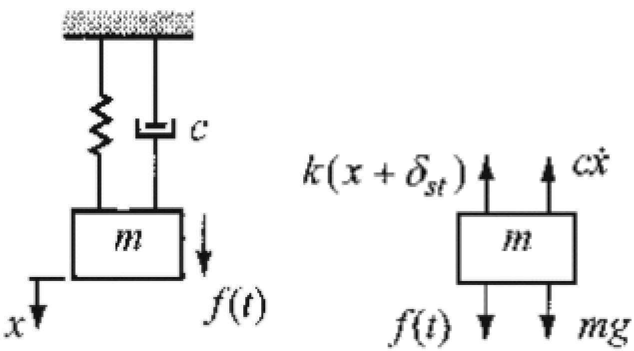

Figure 1 One degree of freedom model

The fundamental physical law governing all vibration phenomena is Newton's second law, which in its most commonly used form says that *the sum of the forces acting upon an object is equal to its mass times its acceleration.* Force and acceleration are both vectors, so Newton's second law, written in its general form, yields a vector equation. For the one degree of freedom (DOF) system, this reduces to a scalar equation, as follows:

$$F = ma \tag{1}$$

Where:

- F – Sum of forces acting upon body;
- m – Mass of body;
- a – body acceleration.

For the system in Figure 1, equation (1) yields its differential equation of motion, as follows:

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{2}$$

The solution for the motion of the unforced one degree of freedom system is important in its own right but specifically important in laying the groundwork to study self excited instability rotor vibrations. If the system is considered to be unforced, then $f_{(t)}=0$ and equation (2) becomes the following:

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{3}$$

This is a second order homogeneous ordinary differential equation. To solve for x $_{(t)}$, from Equation (3), one needs to specify the two initial conditions, $x_{(0)}$ and $\dot{x}_{(0)}$. As summing k and c are both positive, three categories of solution can result from equation (3), (a) under damped, (b) critically damped, and (c) over damped. These are just the traditional labels used to describe the three distinct types of roots and the corresponding three motion categories that equation (3) can potentially yield when k and c are both positive. Substituting the known solution form ($Ce^{\lambda t}$) into equation (3) and then cancelling out the solution form yield the following quadratic equation for its roots (Eigen values) and leads to the equation for the extracted two roots, $\lambda_{1,2}$, as follows:

$$m\lambda^2 + c\lambda + k = 0 \tag{4}$$

$$\lambda_{1,2} = -\frac{c}{2m} \pm \sqrt{(\frac{c}{2m})^2 - (\frac{k}{m})} \tag{5}$$

The three categories of root types possible from equation (4) are listed as follows [3]:

Under damped, $(\frac{c}{2m})^2 < (\frac{k}{m})$, complex conjugate roots, $\lambda_{1,2}=\alpha \pm i\omega_d$;

Critically damped, $(\frac{c}{2m})^2 = (\frac{k}{m})$, equal real roots, $\lambda_{1,2}=\alpha$;

Over damped, $(\frac{c}{2m})^2 > (\frac{k}{m})$, real roots, $\lambda_{1,2}=\alpha \pm \beta$.

4. GEARS AND VIBRATION

Vibration diagnosis is extending number of parameters about present and future condition and quality of component parts in dynamic system (in this case, elements in power transmission system). These parameters give better presentation of system for power transmission without dismantling parts of system.

Conditions for vehicle vibration measurement in exploitation working conditions are of non-steady nature (number of revolution n≠const.).

Vibrations are synchronised emitted with number of revolution of rotating mass. In frequent spectrum prominent characteristics synchronised components of vibrations are being generated – harmonic from kinematics frequency, that make harmonic series (orders).

Connection of rpm and order is:

$$OrderN(Hz) = N \times \frac{RPM}{60} \tag{6}$$

The first order is ratio of RPM and 60, and Nth order is multiplication of Ith order and integer, like in this example: impeller speed = 1200 rpm, I–order is at 20 Hz, and so forth.

Basic order is the first order. But, in gears, basic vibrations are consequence of RPM and gear-wheels. Te first shaft order is one characteristic for gearbox. Second characteristic order is I-order multiplicities with m (m-module of gear-wheel). That is I-order for gear in conjunction.

Potential cause of expanded vibrations can be battered, excenter or damaged gears.

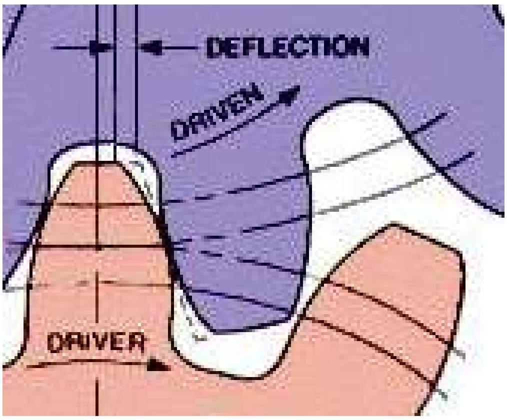

Figure 1 Conjugate gear action

Defect gears generate high and medium frequency vibration, which are suitable to multiplication of defect gear tooth number (r) and RPM of drive gear (n). Signal form from defect gear toot can be recognized.

$$f_{toothing} = f_{shaft} \times N_{tooth} \tag{7}$$

Variable vibrations are occurring in gear-wheels that work under low load, because loading is advancing from one tooth to another.

Beside characteristic frequencies coupled to basic number of revolutions of rotating elements, in spectrum there are also and fix frequencies combined with structural, resonant system characteristics. Impacts, which are results of load changing, will excite gear natural frequency. Frequency analysis has important part in failure prediction at gearbox, and gives us a non-destructive tool for determine time of maintenance in transmission elements.

Gear vibration can be observed like free damped vibrations where excite regenerate itself with every impact in every conjugate gear action next tooth's couple [2].

4. 2 Cepstrum analysis

Cepstrum analysis is a tool for the detection of periodicity in a frequency spectrum, and seems so far to have been used mainly in speech analysis for voice pitch determination and related questions. In that case the periodicity in the spectrum is given by the many harmonics of the fundamental voice frequency, but another form of periodicity can also be detected by cepstrum analysis is the presence of sidebands spaced at equal intervals around one or a number of carrier frequencies. The presence of such sidebands is of interest in the analysis of gearbox vibration signals, since a number of faults tend to cause modulation of the vibration pattern resulting from tooth meshing, and this modulation (either amplitude or frequency modulation) gives rise to sidebands in the frequency spectrum. The sidebands are grouped around the tooth-meshing frequency and its harmonics, spaced at multiples of the modulating frequencies, and determination of these modulation frequencies can be very useful in diagnosis of the fault [4].

The cepstrum exist in various forms but all can be considered as a spectrum of a logarithmic (amplitude) spectrum. This means that it can be used for detection of any periodic structure in spectrum, e.g. from harmonic, sidebands, or the effects of echoes. It is also shown, however, that effects which are convolved in the time signal (multiplied in the spectrum) become additive in the spectrum, and subtraction there results in a deconvolution.

The cepstrum was first proposed as far back as 1963 and was at that time defined as "the power spectrum of the logarithmic power spectrum". The intended application at that time was to seismic signals because it was realised that it would give information about echoes and that in turn would help to determine the depth of the hypocentre of a seismic event. The reason for defining the cepstrum as above is not entirely clear, as even in the original paper it is compared with the autocorrelation function, which can be obtained as the inverse Fourier transform of the power spectrum. Later, another definition of the cepstrum was given as "the inverse Fourier transform of the logarithmic power spectrum", thus making its connection with the autocorrelation clearer. At about the same time, another cepstrum like function was defined as the "inverse Fourier transform of the complex logarithm of the complex spectrum" and to distinguish it from the above cepstra it was called the "complex cepstrum", while they were renamed "power cepstra".

Cepstrum is normally defined as the power spectrum of the logarithm of the power spectrum. Quefrency is the independent variable of the cepstrum and has the dimensions of the time as in the case of the autocorrelation. The quefrency is seconds is the reciprocal of the frequency spacing in Hz in the original frequency spectrum, of a particular periodically repeating component. Just as the frequency in normal spectrum says nothing about absolute time, but only about repeated time intervals (the periodic time), the quefrency only gives information about frequency spacing’s and not about absolute frequency [4].

Using the function terminology to indicate the forward Fourier transform of the bracketed quantity, the original definition of the cepstrum is:

$$c_{(t)} = \left| F\{\log F_{xx}(f)\} \right|^2 \tag{8}$$

Where the power spectrum of the time signal $f_x(t)$ is given by:

$$F_{xx}(f) = \left| F\{f_x * t)\} \right|^2 \tag{9}$$

The new definition of the power cepstrum is:

$$c_p(t) = F^{-1}\{\log F_{xx}(f)\} \tag{10}$$

While the autocorrelation function is given by:

$$R_{xx}(t) = F^{-1}\{F_{xx}(f)\} \tag{11}$$

This can be interpreted as the square root of equation (8) or as the modulus of equation (10), since for real even function such as a log power spectrum, the forward and inverse transforms give the same result. The complex cepstrum may be defined as fallows:

$$c_c(t) = F^{-1}\{\log F_x(f)\} \tag{12}$$

Where $F_x(t)$ is the complex spectrum of $f_x(t)$ i.e.

$$F_x(f) = f\{f_x(t)\} = a_x(f) + ib_x(f) = A_x(f)e^{i\phi_x(f)} \tag{13}$$

In terms of real and imaginary components or amplitude and phase, respectively. From (13) the (complex) logarithm of F_x (f) is given by:

$$\log F_x(f) = \log A_x(f) + i\phi_x(f) \tag{14}$$

Where $f_x(t)$ is real, as is normally the case, then $F_x(f)$ is "conjugate even", i.e.

$$F_x(f) = F_x^*(f) \tag{15}$$

From the general theory of the Fourier transforms it is known that the spectrum should be of real even function is real and even, and of a real odd function is imaginary and odd. Since any real function can be divided into even and odd components it follows that the real part of the Fourier transform comes from the even part of the time signal, and the imaginary part from the odd part of the time signal.

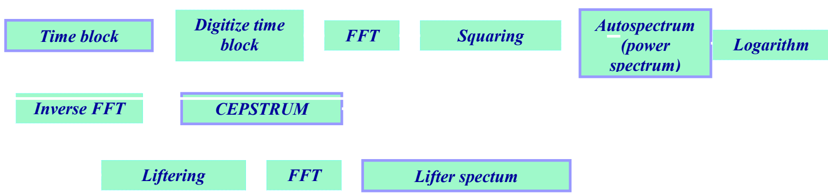

Figure 2 Data processing schemes

For determine period in spectrum and enable research of modulation, for gear set analysis is characteristically Cepstrum tool, that present spectrum in time domain.

5. CASE STUDY OF MEASUREMENT AND VIBRATION MONITORING

The vibration diagnosis and analysis is following steps:

- Analysis dynamic system, which is the object of measurement.
- Preparation for measurement.
- Vibration measurement and number of revolutions.
- Analysis of measured vibration signal (frequency and harmonic).
- Diagnostic conclusion.

Measurement object was gear set of vehicle, which is a prototype (off-road vehicle with regular and non planetary gearbox). All measurements have been made in mountain off-road conditions, in every gear. For measurement we used four acceleration transducers, CTC–131 type. There was four points for measurement. Precise measurement points were:

- Two points in input shaft (transducers 1 and 3), in place of input bearing.
- Two in output shaft (transducers 2 and 4), in place of output bearing.

Multi channel measurement, type NETdB 12, French manufacturer 01 dB Metravib, Areva Corporation. Measurement equipment is set with software also from 01-dB Metravib, Areva Corporation. Measurement place for transducers were bearing boxes of input and output shift. Measurement set signal was recorded to PC hard-disk (laptop), for future analysis. All measurements are being made at about 2000 rpm of vehicle engine, with calculation of transmission ratio.

In the beginning, nought state was measured, for determing starting parameters for measurement. After, only periodically measurements are being made (at about every 2000 km). Next parameters were measured:

- Absolute vibrations with four accelerometers in measurement points.
- RPM with laser tachometer.

Measurements were in real-time, what enable various on-line testing.

With post-process mathematic averaging we are processing random time signal, which we recorded in measurement of working load characteristics (vibration and number of revolutions) in off-road conditions drive. After all measurements, we made vibration analysis, which include:

- Time signal filtration.
- Recorded acceleration signal integration.
- FFT (Fast Fourier Transformation) transformation.
- Mathematic parameter selection (RMS, log) for presenting 2D I 3D (Waterfall) spectrum for analysis.
- Cepstrum analysis.

For gear vibration signal, magnitude modulation is characteristic, as a result of excenter and frequency modulation appearance, which cause variation of some gear wheel rpm. There are several characteristics for gear vibrations:

- Frequencies from errors in making gears, that are nonsensitive to load alteration.
- Sidebands from magnitude (tooth conjuction deformation) and frequency (rpm fluctuation) modulation.
- Low harmonics from extra impacts, at every rpm.
- Combination of frequency components of conjunction.

For this measurement we used laser tacho probe too, made by Monarch instruments, attached to gearbox output shaft. From numerous measurements we made, in paper we'll present spectrum for second gear in different off-road conditions, for transducer 4.

6. SECOND GEAR

All measurements were made in real off-road conditions, in different off-road types. We made measurements in off-road conditions at road at mountain Fruska gora and Zlatibor. Route and ground quality of road was very similar in both test roads. All the measurements were made in uphill driving, so the ground loading was the same in both ways.

6. 3 Second gear at Fruska gora

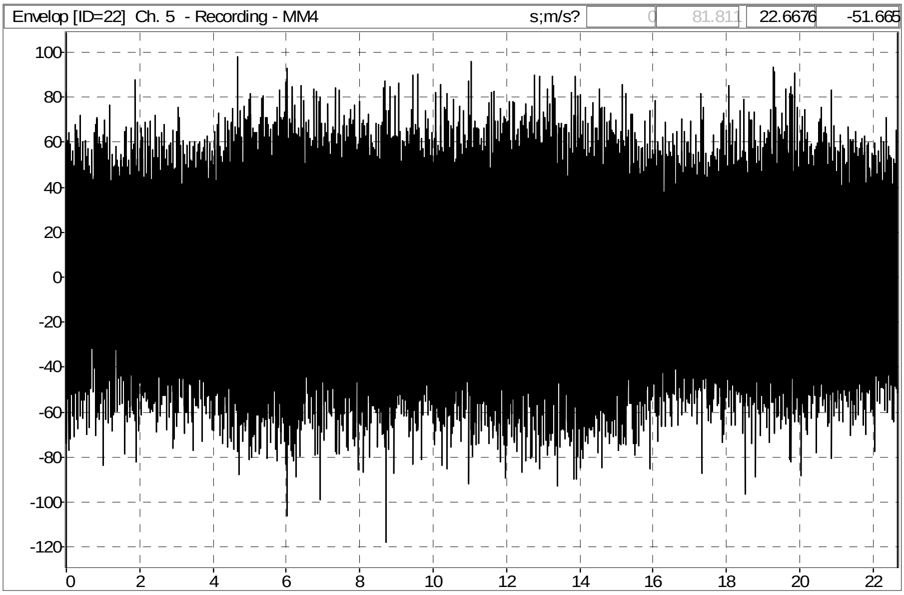

Figure 3 *Speed signal in time for the second gear, Fruska gora (m/s2 / s)*

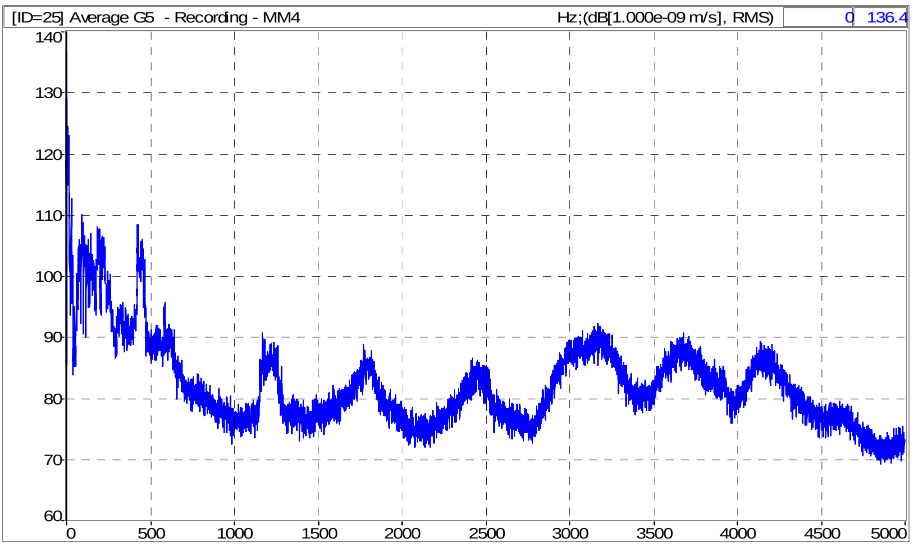

Figure 4 *Frequency spectrum, Fruska gora (dB/Hz)*

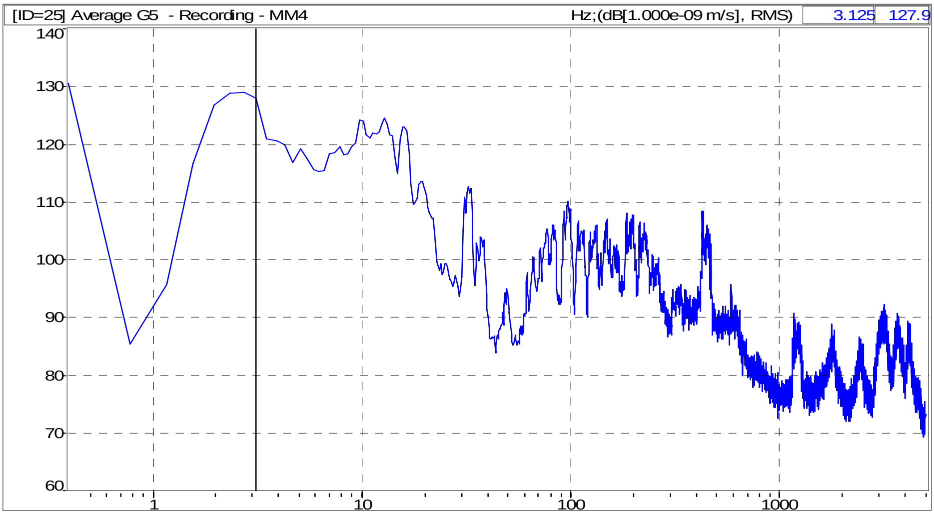

Figure 5 *Logarithm of the frequency spectrum, Fruska gora (dB / Hz)*

Frequency spectrum is giving us the levels of vibrations, picks and frequencies that are characteristic for some pick values. In frequency spectrum we can identify frequencies from output shaft, conjucted gears, gear mesh frequency and found some frequencies that are not expect to see in that signal. Logarithm of frequency spectrum is magnifaing the beggining of the spectrum, where are some working frequencies of the output shaft and it subharmonics.

Signal of speed (measured in m/s) in time (measured in s) is shown at figure 3. We can see that speed level for gear is from about $\pm$ 60 [m/s] (with some picks from about $\pm$80 [m/s]). The whole frequency spectrum of the time signal in fig. 3 is shown in fig. 4, and the logarithm of frequency spectrum is shown in figure 5. At figure 6, there are shown characteristic frequencies for conjucted output gear, with their subharmonics shown in figure 7 and 8.

Characteristic frequencies are:

- 6,4 Hz, frequency of the I-order of the output shaft (RPM of the output shaft is at about 386 rpm/60= 6,4 Hz).
- 15,4 Hz, gear mesh frequency of the output shaft and conjugated gears. All subharmonics we can get by multiplication of the first order.
- 554 Hz, the I-order of the conjugate output gear.
- Other subharmonics we can get by multiplication of the first order of output shaft frequency and gear mode.

Only time signal, with subharmonics from specific frequencies is the beginning of the signal analysis.

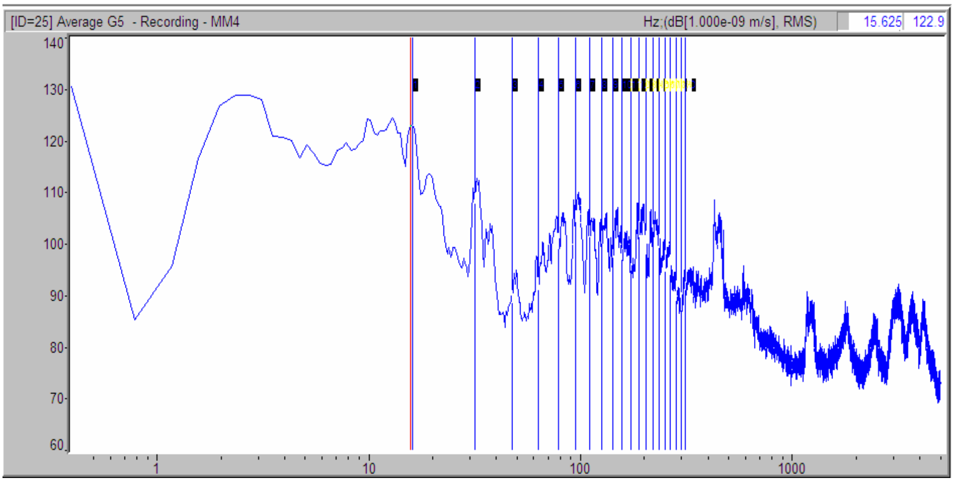

Figure 6 Logarithm of the frequency spectrum, Fruska gora specific frequencies (dB / H)

Display spectrum PUTNI DRUGI UZ.cmg		ID=25
FFT resolution	0.391 Hz	
Harmonic	Frequency	Level
1	15.6	122.9
2	31.3	109.4
3	46.9	88.9
4	62.5	94.3
5	78.1	100.7
6	93.8	103.4
7	109.4	101.6
8	125.0	99.5
9	140.6	95.5
10	156.3	97.3
11	171.9	96.0
12	187.5	102.9
13	203.1	101.9
14	218.8	102.4
15	234.4	100.0
16	250.0	96.3
17	265.6	97.2
18	281.3	91.9
19	296.9	88.0
20	312.5	91.2

Display spectrum PUTNI DRUGI UZ.cmg		ID=25
FFT resolution	0.391 Hz	
Harmonic	Frequency	Level
1	3.1	127.9
2	6.3	115.2
3	9.4	120.1
4	12.5	123.4
5	15.6	122.9
6	18.8	113.0
7	21.9	107.2
8	25.0	99.3
9	28.1	97.1
10	31.3	109.4
11	34.4	98.2
12	37.5	103.7
13	40.6	88.0
14	43.8	83.7
15	46.9	88.9
16	50.0	94.1
17	53.1	85.2
18	56.3	85.9
19	59.4	87.3
20	62.5	94.3

Figure 7 Subharmonic of the first gear mesh and the output shaft

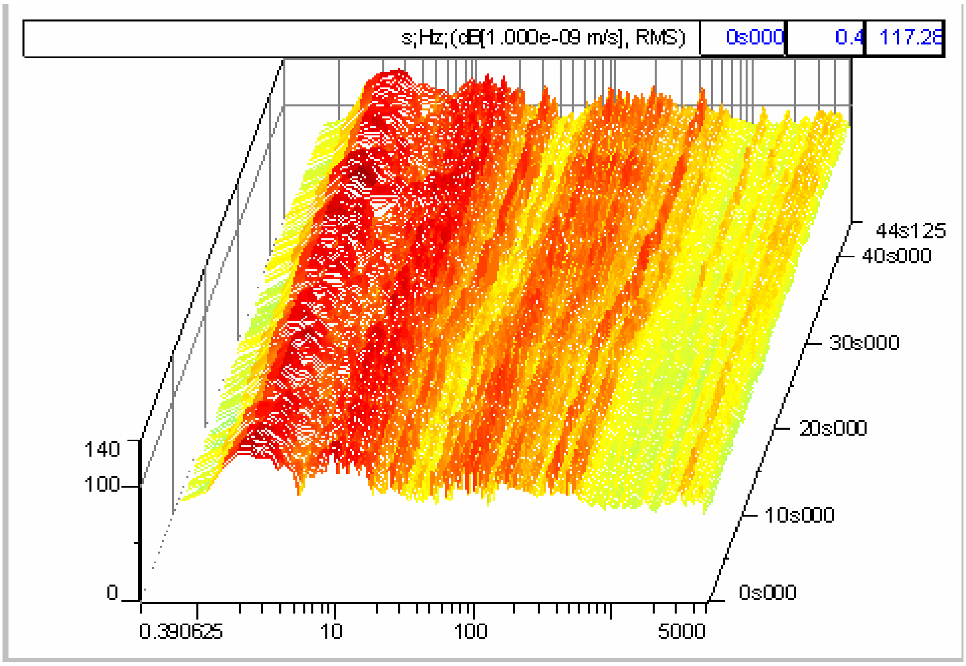

Figure 8 3D spectrums for second gear, Fruska gora (dB / Hz / s)

In 3d spectrum diagram we can see specific frequencies, in multispectrum conditions. Time, frequency and pick levels are shown in one diagram.

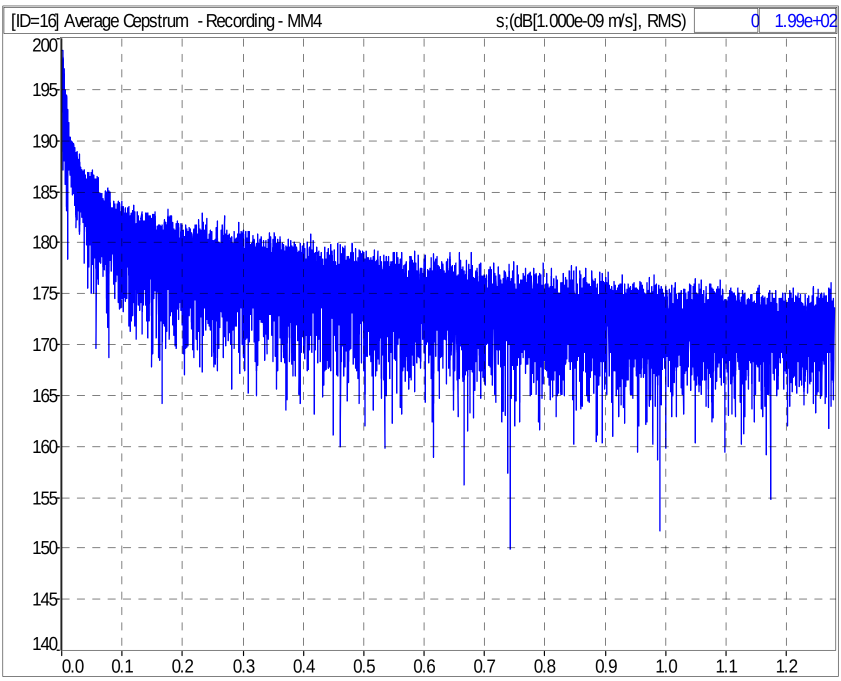

Figure 9 Cepstrum of recorded signal (dB / s)

Cepstrum of the signal of vibration measured in second gear of transmission at Fruska gora off-road conditions is shown in figure 9. The main advantage of cepstum is, because he's finding all periodic events (it is shown as diagram pick at some time). In this signal there was no characteristics picks at any time. In this signal we can see that there is no pick values that would made some suspicion that something is at (or will be in short time period) failure. The conclusion was that in that gear there is no specific problem with gears in second gear.

6. 4 Second gear at Zlatibor

Signal of speed (measured in [m/s]) in time (measured in s) is shown at figure 10, for the off-road road recorded signal in Zlatibor. We can see that speed level for gear is from about ± 60 [m/s] (with some picks from about ±80[m/s]). The whole frequency spectrum of the time signal in fig. 10 is shown in fig. 11, and the logarithm of frequency spectrum is shown in figure 12.

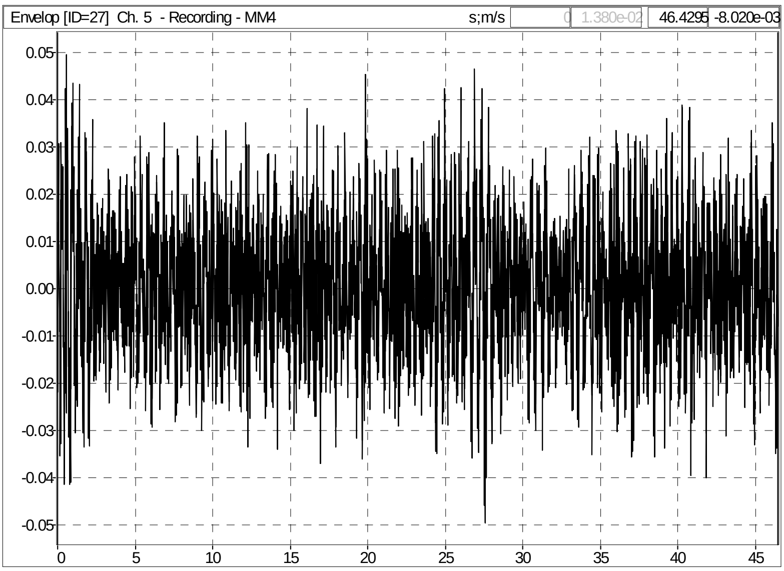

Figure 10 Speed signal in time, Zlatibor (m/s / s)

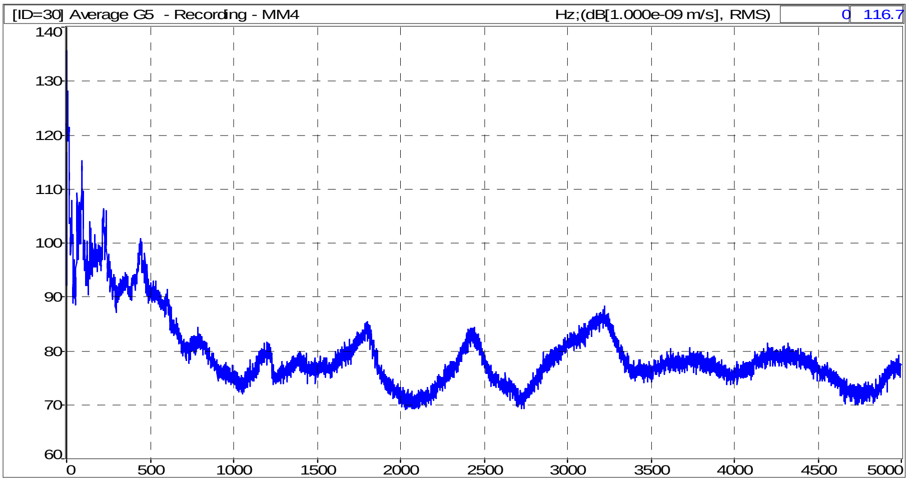

Figure 11 Frequency spectrum, Zlatibor (dB / Hz)

Characteristic frequencies are the same like in Fruska gora off-road road:

- 6,4 Hz, frequency of the I-order of the output shaft (RPM of the output shaft is at about 386 rpm/60= 6,4 Hz).
- 15,4 Hz, gear mesh frequency of the output shaft and conjugated gears. All subharmonics we can get by multiplication of the first order.
- 554 Hz, the I-order of the conjugate output gear.
- Other subharmonics we can get by multiplication of the first order of output shaft frequency and gear mode.

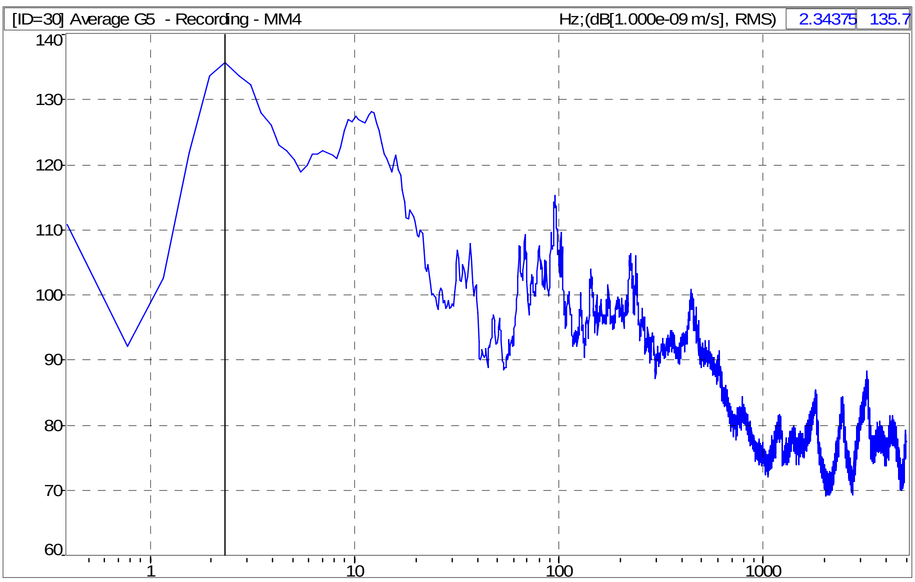

Figure 12 Logarithm of the frequency spectrum, Zlatibor (dB / Hz)

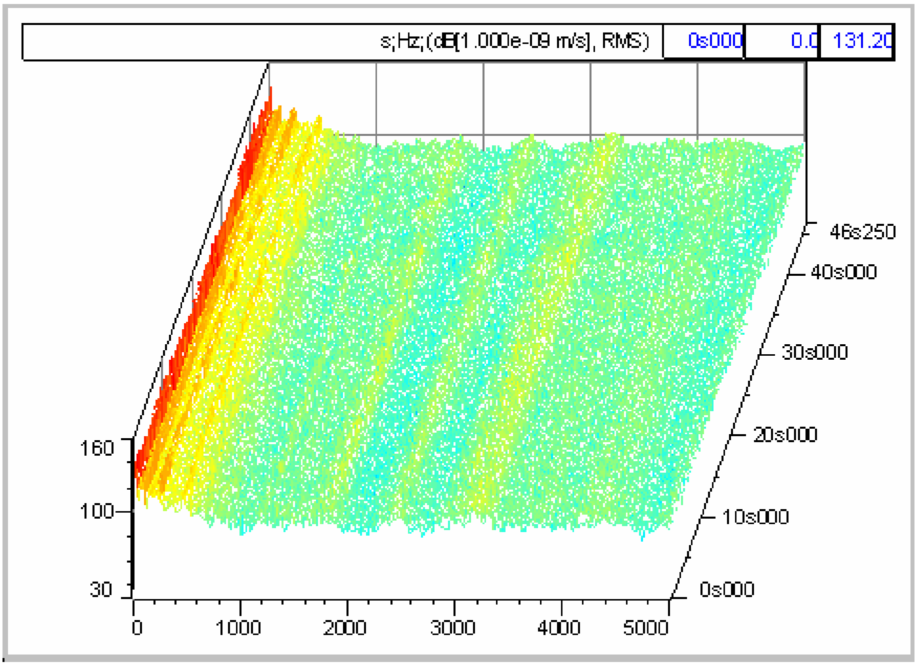

Figure 13 3D spectrum for second gear, Zlatibor (dB / Hz / s)

At figure 13 we can see 3D spectrum of measured vibration signal. If we make some comparation with 3d spectrum in Fruska gora, we can see that "setttling" of the spectrum, as the vehicle is being tested in time. The reason of the spectrum "setttling" is running in gears in transmission with time period of working. The gears are being "more conjucted" in working time, and that is shown in figure 8 and 13.

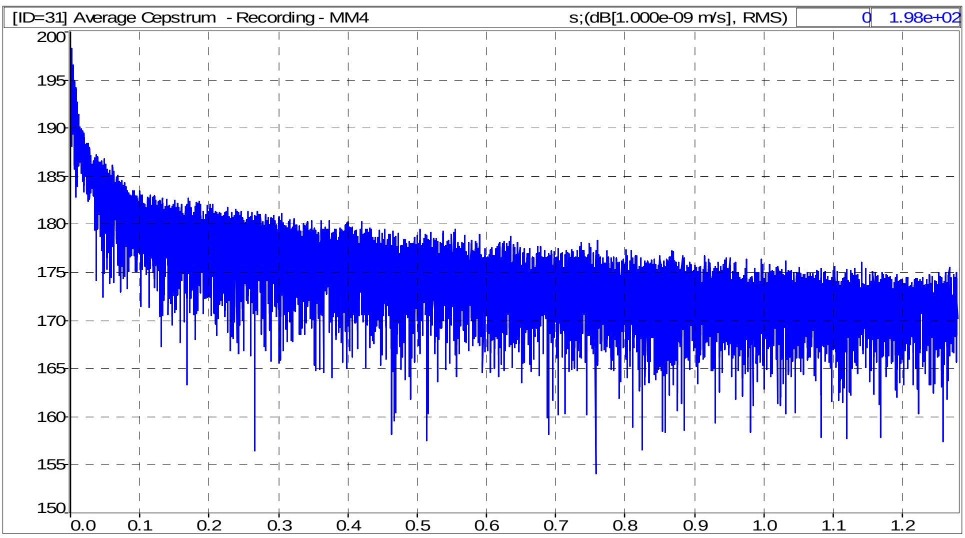

Figure 14 Cepstrum of recorded signal (dB / s)

Cepstrum of the recorded signal is shown in figure 14. Cepstum is finding all periodic events and showing it at diagram like a pick at some time. In this signal there was no characteristics picks at any time. The conclusion was that in that gear there is no specific problem with gears in second gear.

7. CONCLUSIONS

Cepstum tool for vibration analysis is finding all periodic events and showing it at diagram, like a pick at some time. It is wary suitable for non aggressive gear testing and condition analysis in real working conditions of vehicle transmission.

In our signals in second gear, that we recorded during vehicle testing in two off-road conditions in same conditions of testing we found no characteristics picks at any time. Main conclusion was that gear set is working without damages or failures that are going too happened. The measurements are being made in the beginning of the testing, at Fruska gora, and in the middle of testing, at Zlatibor. At the end of testing, we opened gear set and explore all gears in gear set of vehicle transmission. We didn’t found out any failure. That gave our conclusions a form of evaluation.

The second characteristic of transmission that we can see is "setttling" in 3D spectrum, in time of testing. The 3D spectrum in Zlatibor, being measured in the middle of testing, is more "setttled" than the 3D spectrum recorded at Fruska gora, in the beginning of vehicle testing. Conclusion was, reason of the spectrum "setttling" is running in gears in transmission with time period of working (the gears are being "more conjucted").

Usage of cepstrum analysis in vibration analysis of vehicle gear transmission testing in validation and estimation of working conditions and failures (future failures) is a relative new approach in vehicle testing. With respect to it non damage nature, in future time it`s going to be more exploit tool in vehicle testing, and maintenance and monitoring of working condition in vehicle work time period. We can detect weak spots in early phase, without dismantling of transmission. All the testing we can make faster and more economic.

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