NONLINEAR DYNAMICS OF HEAVY GYRO ROTORS

Ljiljana Veljović¹, PhD, Assistant professor, Dragan Milosavljević, PhD, Full professor, Gordana Bogdanović, PhD, Assistant professor, Aleksandar Radaković, MSc, teachnig assistant, Mirjana Lazić, PhD, Assistant professor

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INTRODUCTION

Heavy bodies in motion, artillery projectiles in motion, rotors of turbines, different mobile installations on ships, aircraft propeller rotating possess special properties known as properties of gyroscopes. Tern or a toy top is just a simple well-known toy with unusual property that when it rotates by high angular velocity about its axis of symmetry, it keeps in the state of stationary rotation around this axis. This feature has attracted scientists around the world and as a result of year's research many devices and instruments are created; from simple to very complex structures, which operate on the principle of a spinning top that plays an important role in stabilizing the movement. Ability gyroscope that keeps the line was used in many fields of mechanical engineering, mining, aviation, navigation, military industry and in celestial mechanics. Gyroscopes are used to measure angular rotation rate in airplanes, spacecrafts, missiles, automobiles and even consumer electronics.

Gyroscope's name comes from the Greek words $\gamma \nu \rho \rho$ (turn) and $\sigma \kappa \sigma \pi \epsilon \omega$ (observed) and is related to the experiments that the 1852nd were painted by Jean Bernard Leon Foucault. The principle of gyroscope based on the principle of pseudo regular precession. Gyroscopes are very responsible parts of instruments for aircraft, rockets, missiles, transport vehicles, many weapons and robotics. Gyroscopes are used in several forms in transportation. Car engines act like big gyroscopes, so the racing industry takes them into consideration. Because the cars only turn in one direction, the gyroscopic force helps the car stay on the track. Motorcycle wheels also act as gyroscopes to make the bike easier to balance. Gyroscopic behaviour is used in the racing car industry because car engines act just like big gyroscopes. This has its uses, for example in the American Indy car racing. During the race the cars go round the circuit in one direction only. Because of the gyroscopic forces from the engine depending on whether the engine is spinning closewise or anti-clockwise the cars nose will be forced up or down. Providing the engine spins in the right direction it can help the car to stay on the track.

Gyroscopes are also used in monorails and ships to help them stay upright [17]. They are used in navigation to stabilize the movement of ships in a seaway, to change direction, and direction of angular and translator velocity projectiles, and in many other special purposes. Mathematical analysis of the two-wheeled vehicle gyroscopic stabilization problem first appears in [2], and more recently in [17], without derivation, or in [14], where the derivation is by use of bond graphs. The problem of gyropscopic stabilization of unstable vehicles in roll is considered in [17] using Lagrangian dynamics. Linearized versions of the equations of motion show that the stability conditions are dependent on turn rate and direction for the single gyro case, but not for the double gyro case.

¹ Ljiljana Veljović, University of Kragujevac, Faculty of Engineering, Sestre Janjić 6, 34000 Kragujevac, Serbia, veljovicljilja@yahoo.co.uk

Majority of emerging safety and comfort system require yaw- and roll-rate sensing.Yaw-rate sensors for electronic stability control (ESC) brake systems, which is a major breakthrough in automotive safety have been the most significant. By automatically activating asymmetric individual wheel braking actions in an out-of-control vehicle to stabilize it to regain traction and driver control ESC helps prevent accidents. Automotive applications are known to impose quite harsh environmental conditions such as vibration, shock, temperature, and thermal cycling on inertial sensors [18]. Micro machined gyroscopes are known to be especially challenging to develop and commercialize due to high sensitivity of their dynamic response to fabrication and environmental variations. Meeting performance specifications in the demanding automotive environment with low-cost and high-yield devices requires a very robust microelectromechanical systems (MEMS) sensing element. [20].

There are many devices that are applied to the military, and their design is based on the principles of gyroscopes. This gives them a very important role, and they need to be under the strict control of the design because in case of damage they could lead to catastrophic consequences. High-yield devices require very robust microelectromechanical systems (MEMS) sensing element. MEMS angular rate sensors have implicated in the automotive field since the mid-1990s (yaw-rate sensors for electronic stability control, rollrate gyroscopes to sense impending vehicle rollover conditions) [5]. Gyroscopes in lanekeeping systems assist the drivers steering action to preserve vehicle in existing highway lane.

Gyroscope is a homogeneous, axis-symmetric rotating body that rotates by large angular velocity about its axis of symmetry. It is one of the most inertial sensors that measure angular velocity and small (angular disturbances) angular displacements around the reference axis.

Technical applications gyros today are so manifold and diverse that there is a need to get out of the general theory of gyroscopes allocates a separate discipline, called "applied theory of gyroscopes."

The online Museum of Retro Technology [19] cites many articles and examples of gyrocars, including a 1961 Ford Gyrocar concept called the Gyron and a concept from Gyro Transport Systems of Northridge, California that was on the cover of the September, 1967 issue of "Science and Mechanics".

Each mechanical gyroscope is based on coupled rotations around more axes with one point intersection. Most of the old equipment was based on rotation of complex and coupled component rotations which resulting in rotation about fixed point gyroscopes. The classical book [1] by Andonov et al. contains a classical and very important elementary dynamical model of the simple case of the gyrorotor, and presents an analogous and useful dynamical and mathematical model of nonlinear dynamics [21, 22]

This work is different in that we derive the equations of motion using vector method proposed by K. Hedrih [9], and propose stability analysis for the system based on the derived model.

The vector approach is very suitable to obtain new view to the properties of dynamics of pure classical task, investigated by numerous generations of the researchers and serious scientists around the world.

Using Hedrih's (See [5-10]) mass moment vectors some characteristics members of the vector expressions of derivatives of linear momentum and angular momentum for the gyrorotor coupled rotations around two axes without intersection obtain physical and dynamical visible properties of the complex system dynamics. Between them there are

vector terms that present deviation couple effect containing vector rotators whose directions are the same as kinetic pressure components on corresponding gyrorotor shaft bearings.

Organizations of this paper based on the vector method applications with use of the mass moment vectors and vector rotators for obtaining vector expressions for linear momentum and angular momentum and their derivatives of the rigid body coupled rotations around two axes without intersections. These obtained expressions are analyzed and series of conclusions are pointed out, all useful for analysis of the rigid body coupled rotations around two axes without intersections when system dynamics is with two degrees of mobility as well as with two degrees of freedom, or for constrained by programmed rheonomic constraint and with one degree of freedom. It is possible to obtain two nonlinear differential equations in scalar form for rotations about each axes and also corresponding kinetic pressures in vector form by using two vector equations of dynamic equilibrium of rigid body dynamics with coupled rotations around two axes without intersections around two axes without intersections for two degrees of freedom.

This paper presents a new concept of mechanical design gyroscope. It is expected that multi-DOF concept will lead to reliable, robust, and high performance-angular-rate sensors with high yields, ideal for the demanding automotive environment.

1. MATHEMATICAL MODELING

A number of researchers have devised a mathematical model of a rotor based on the classical engineering problems so it takes place in world scientific and engineering professional literature [21,22]. Many scientists have studied disc as a mathematical model for rotor. Problem of dynamics of eccentricity, skew positioned disc on the shaft rotation is classical problem with gyroscopic effect which take place in all text books of Dynamics and Theory of Oscillations with application in engineering. But, their presentation was finished only by nonlinear differential equations without solutions and expressions for kinetic pressure. Simes, Stodola et al. [4] linearized nonlinear dynamics problems and got some results. It was like they used simplified model. The use of the simplified model could limit the ultimate performance of the designed controller, especially if more aggressive or fast maneuvers are desired.

We will model the gyrorotor as a rigid eccentric disc (eccentricity is *e* with mass *m* and radius *r* which is inclined to the axes of its own rotation by the angle β . Although this is not physically true, this assumption helps us to collect all the various inertia effects such as main and tail rotor dynamics into a simple constant mass-inertia matrix. The angle of own rotation around moveable axis oriented by the unit vector \vec{n}_1 is φ_1 and the angular velocity is $\vec{\omega}_1$. The angle of rotation around the shaft support axis oriented by the unit vector \vec{n}_2 is φ_2 and the angular velocity is φ_2 (see Figure 1 a*). When the support shaft is vertical and the rotor shaft is horizontal and when they are without intersection, with orthogonal distance *a* we obtain that angular velocity of rotor is $\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2$ and in a form $\vec{\omega} = \omega_1 \vec{n}_1 + \omega_2 \vec{n}_2$.



Figure 1 Model of gyrorotor

1.1 Coordinates and Frames

As shown in Figure 1, there are two frames that we consider: the reference support frame and the body frame attached to the disc. The transformation between the two frames is given by the homogeneous transformation matrix where the rotation matrix $_0 R^1$ represents the relative orientation between the two frames. The rotation matrix can be expressed in coordinates using yaw-pitch-roll Euler angles as:

$${}_{0}R^{1} = \begin{bmatrix} \cos\varphi_{2}\cos\varphi_{1} & \cos\varphi_{2}\sin\varphi_{1} & \sin\varphi_{2} \\ -\sin\varphi_{2}\cos\varphi_{1} & -\sin\varphi_{2}\sin\varphi_{1} & \cos\varphi_{2} \\ \sin\varphi_{1} & -\cos\varphi_{1} & 0 \end{bmatrix}$$
(1)

1.2 Rigid body equations

The angles φ_1 and φ_2 are generalized coordinates in case when we investigate system with two degrees of freedom. In this case φ_1 is generalized coordinate. The second angle φ_2 is a rheonomic coordinate which is defined by a time function. The tensor matrix of mass inertia moments of eccentric disc in a relation to the point O, (this point is cross section of rotor support axis and a plane which contains the support shaft axis and that is perpendicular to rotor shaft axis as it can be seen in (Fig.1), in the system coordinate axes coupled with support.

Velocity expression for elementary body mass particle is in a form $\vec{v} = [\vec{\omega}_2, \vec{r}_0] + [\vec{\omega}_1 + \vec{\omega}_2, \vec{\rho}]$, so by using basic definitions for linear momentum and angular momentum we can obtain main expressions in aim to get differential equations.

Vector expression for linear momentum is:

$$\vec{K} = \left[\vec{\omega}_2, \vec{r}_0\right] M + \vec{\omega}_1 \vec{S}_{\vec{n}_1}^{(O_1)} + \vec{\omega}_2 \vec{S}_{\vec{n}_2}^{(O_1)}$$
(2)

The angular momentum of the system is:

$$\vec{L}_{O_1} = \omega_2 \vec{n}_2 r_0^2 M + \omega_2 \left[\vec{\rho}_C, \left[\vec{n}_2, \vec{r}_0 \right] \right] M + \omega_1 \left[\vec{r}_0, \vec{S}_{\vec{n}_1}^{(O_1)} \right] + \omega_2 \left[\vec{r}_0, \vec{S}_{\vec{n}_2}^{(O_1)} \right] + \omega_1 \vec{J}_{\vec{n}_1}^{(O_1)} + \omega_2 \vec{J}_{\vec{n}_2}^{(O_1)}$$
(3)

Where
$$\vec{S}_{\vec{n}_1}^{(O_1)} = \iiint_V [\vec{n}_1, \vec{\rho}] dm$$
 and $\vec{S}_{\vec{n}_2}^{(O_1)} = \iiint_V [\vec{n}_2, \vec{\rho}] dm$, $dm = \sigma dV$ are corresponding

body mass linear moments of the rigid body for the axes oriented by direction of component angular velocities of coupled rotations through the moveable pole O_1 on self rotating axis and $\vec{J}_{\vec{n}_1}^{(O_1)} = \iiint_V [\vec{\rho}, [\vec{n}_1, \vec{\rho}]] dm$ and $\vec{J}_{\vec{n}_2}^{(O_1)} = \iiint_V [\vec{\rho}, [\vec{n}_2, \vec{\rho}]] dm$ are corresponding body mass

inertia moments of the rigid body for the axes oriented by direction of component angular velocities of coupled rotations through the moveable pole O_1 on self rotating axis.

We apply first derivative of angular momentum in the form

$$\frac{d\hat{L}_{O_{1}}}{dt} = \vec{\omega}_{2}r_{0}^{2}M + \omega_{1}\omega_{2}\left\{\left[\vec{n}_{1},\vec{J}_{\vec{n}_{2}}^{(O_{1})}\right] + \left[\vec{n}_{2},\vec{J}_{\vec{n}_{1}}^{(O_{1})}\right] + \left[\vec{n}_{2},\vec{J}_{\vec{$$

The dynamical equations for motion on a straight inclined track can be easily obtained from the theorem of angular momentum derivative which in this case is in a form [16]:

$$\frac{dL_{O_1}}{dt} = \vec{M}_{O_1} \left(\vec{F}_i\right) - \left[\!\left[\vec{\omega}_2, \vec{r}_0\right]\!\right]\!\left[\vec{\omega}_1 + \vec{\omega}_2, \vec{\rho}_C\right]\!\right]\!M$$
(5)

This leads to the two vector equations of rigid body coupled rotation around axes without intersection [15,16]. These vector expressions can be used in general case when considered system has two degree of freedom. In case when the second angle φ_2 is a rheonomic coordinate which is defined by a time function $\varphi_2 = \omega_2 t + \varphi_{20}$, we have vector equation in a form:

$$\ddot{\varphi}_{1} + \Omega^{2} (\lambda - \cos \varphi_{1}) \sin \varphi_{1} + \Omega^{2} \psi \cos \varphi_{1} = 0$$
(6)

where constants in differential equation are in following form:

$$\Omega^{2} = \omega_{2}^{2} \frac{\varepsilon \sin^{2} \beta - 1}{\varepsilon \sin^{2} \beta + 1}, \qquad \varepsilon = 1 + \left(\frac{2e}{r}\right)^{2}, \qquad \lambda = \frac{g(\varepsilon - 1)\sin\beta}{e\omega_{2}^{2}(\varepsilon \sin^{2} \beta - 1)},$$

$$\psi = \frac{2ea\sin\beta}{er(\varepsilon \sin^{2} \beta - 1)}$$
(7)

Transforming previous nonlinear differential equation into system of two first order nonlinear differential equations it is possible to obtain stationary values which correspond to the relative equilibrium positions of the disc on the self rotating axis. For each of relative equilibrium position Lyapunov criteria of stability can be applied and then it can be concluded about center or saddle points.

2. PHASE PORTRAIT OF THE GYROROTOR

Relative nonlinear dynamics of heavy gyrorotor disc around self rotation shaft axis is possible to present by means of phase portrait method. Phase trajectories forms and their transformations by changing of initial conditions and for different cases of disc eccentricity express nonlinear phenomena.

The solution-first integral of differential equation (6) with the initial conditions $t_0 = 0$, $\varphi_1(t_0) = \varphi_{10}$, $\dot{\varphi}_1(t_0) = \dot{\varphi}_{10}$, we obtain in the following form:

$$\varphi_{1}^{2} = \varphi_{10}^{2} + 2\Omega^{2} \bigg(\lambda \cos \varphi_{1} - \frac{1}{2} \cos^{2} \varphi_{1} + \psi \sin \varphi_{1} \bigg) - 2\Omega^{2} \bigg(\lambda \cos \varphi_{10} - \frac{1}{2} \cos^{2} \varphi_{10} + \psi \sin \varphi_{10} \bigg)$$
(8)

and it is the energy integral because the conservative system is analyzed [14-17].

$$\widetilde{E}_{P} = \Omega^{2} \left(\varepsilon \left(\lambda(\varepsilon) (\cos \varphi_{1} - \cos \varphi_{10}) + \frac{1}{2} (\cos^{2} \varphi_{10} - \cos^{2} \varphi_{1}) + \psi(\varepsilon) (\sin \varphi_{1} - \sin \varphi_{01}) \right)$$
(9)

Using MathCAD program on accomplished numerical experiment for researching of existence, like as number and character of stationary values of potential energy, as number of configuration of equilibrium positions and character of their stability, and transformations of phase trajectories with exchanging one of the kinematic parameter of system, graphs of potential energy exchange of corresponding basic system are obtain on numerical way. So, it was very interesting to analyze the influence of these parameter on nonlinear dynamics behavior of system. The potential energy exchange curves for different values of the system parameters (eccentricity e) are given on Figure 2.





Figure 2 Potential energy curves of system

In the case when the dynamic relative equilibrium position is defined by φ_{10} the relative equilibrium position can be stable or unstable and the motion around that position may be defined as the sum of homogenous and particular solution. The character of stability depends on systems parameters.



Figure 3 Stable and unstable relative equilibrium position



Figure 4 Phase trajectories for different system parameters

3. NUMERICAL EXPERIMENT AND GRAPHICAL PRESENTATION

In Figure 4. we can see characteristic homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the dynamic model. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities are shown. The homoclinic trajectories in the form of the number eight are presented in Figure, as well as in the form of the duplicate number eight.

In Figure 4. we can see characteristic phase trajectories portraits for examples of the potential energy curves from Figure 2, and corresponding homoclinic separatrix phase

for different parameters values of the basic system correspond to the gyrorotor dynamic model. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities are presented on Figure 3. We can see more than five types of characteristic phase portraits which contains two types of singular points: by type stable center and unstable saddle

In Figure 4. transformations and layering of the homoclinic trajectories with change of the kinetic parameters values of the basic system correspond to the dynamic model are presented. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities and homoclinic trajectories in the form of the duplicate number eight are, also, presented.

Characteristic potential energy curves and corresponding homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the dynamic model. These examples illustrate of the trigger of the coupled singularities and coupled triggers of the coupled singularities and homoclinic trajectories in the form of the number eight and also in the form of the duplicate number eight.

Figures 3 and 4. for different system parameters, we see that structures of phase portraits are different by types of phase trajectories and homoclinic orbits (phase trajectories of separatrix).

We can see on Figure 4, one-sided separatrix, which are "prolating", and we see also open phase trajectories, which are comprising enclosed phase trajectories which are matching to the periodical oscillator motion-rotations system round stability configurations of equilibrium positions for specific initial conditions when initial angular velocity are small and small angles elongation of rotations, and when that condition are satisfying for any time.

In Figure 4. on phase portrait we notice augmentation of singular points, and we deduce by researching that for some kinetic parameters of system one stable equilibrium position loses stability and that positions now on phase portrait response to homoclinic point by type unstable saddle, but in symmetrical neighborhood appear two near-by stable equilibrium positions (configuration of masses), which on phase portrait response two singular points by center type. We can see also that all of three points are coupled in one "trigger" (trigger of coupled singularities, see reference [6] or [8]). Two stable singular points by type centers enclose one, and the new, closed homoclinic orbit which goes around three singularities, and passing trough one homoclinic point by type saddle in which it selfcross, that it is shaped like form of the number eight or in the form of duplicate of number eight or multiplication. Inside that new separatrix trajectory homoclinical orbit we notice a series of common closed phase trajectories which los instability relative equilibrium positions or relative rest positions, which correspond to periodic oscillatory motion for certain initial conditions, apropos oscillations around new stable position of equilibrium. We notice that homoclinic orbit shaped by number eight and multiplicities of number eight selfcross in points type by saddles which are issue from stable points type by saddle which is lose stability exchanging parameters of system an it is "disintegrate" on three, or even number which are trigger of coupled singularities or coupled triggers of coupled singularities. That point(s) is (are) also bifurcation point(s), because types of bifurcation, and define triple point. In Figure 5 for different system parameters it can be seen how the elongation changes with time.

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Figure 5 Diagrams of elongacy-time

4. CONCLUSIONS

We consider the problem of gyropscopic coupled rotation. We derive the full nonlinear equations of motion for the non-trivial case (coupled rotation around two orthogonal axes without intersection) using Hedrih's vector method dynamics, consider different configurations (different values of eccentricity), and derive numerical versions of the equations of motion. We consider nonlinear dynamics of gyrorotor which are dependent on system parameters. Here we take account only eccentricity but we can consider how another system parameters (angle of inclination and orthogonal distance between axes) influence at nonlinear dynamics of gyrorotor. Also, by using equations of phase trajectories some properties of nonlinearity are investigated. We analyzed homoclinic orbits and their transformation shaped by number eight, their appearance and disappearance by changing someone parameter of system. This is also verified by numerical experiment. Many applications of the discovered vector method by using mass moment vectors are presented for to express kinetic parameters of heavy rotors dynamics as well as of coupled multistep rotors dynamics and for gyrorotors dynamics.

Automotive gyroscopes are highly engineered inertial sensors, involving a large number of disciplines. Stability and robustness of the mechanical sensing element becomes crucial so to improve it multi-DOF sense systems are analysed. It is expected that multi-DOF concept will lead to reliable, robust and high performance angular rate sensors with low production costs and high yields that is ideal for demanding automotive environment. The automotive environment is a daunting combination of temperature, vibration, thermal cycling, shock, humidity, acoustic, etc. Gyroscopes typically operate in safety systems so detection of sensor failures is of utmost importance to avoid an unintended vehicle action that endangers people.

Future work includes further analysis of the equations of motion, and comparisons to other gyroscopic systems such as automotive gyroscope or ship stabilizers. In addition, we will also further analyze control properties, compare the performance of the early mechanical feedback systems to more modern approaches, perform simulations for a full scale vehicle, and analyze results from the scaled model experiments. We can't forget that automotive gyroscope must meet many requirements in addition to having extremely small size and low-unit production cost.

Gyroscopes are simply devices which measure rotation. For robotic applications the gyro output can be used to determine rotation rate, altitude or heading and can be combined with other sensor inputs to determine position. A wide range of robots and autonomous vehicles are currently used in fibre gyros, and many more are likely to be used in the future.

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