DETERMINATION OF PARAMETERS OF THE WEIBULL DISTRIBUTION BY APPLYING THE METHOD OF LEAST SQUARES

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INTRODUCTION

Swedish scientist, Waloddi Weibull, conducted a series of tests, while researching dynamic durability of materials. When he conducted statistical analysis of gained data, he had found that normal distribution could not be used for modelling of statistical features. By generalisation of exponential distribution and adjustment of mathematical model according to empirical distribution, Weibull had reached a new distribution that today bears his name [1, 2].

Undoubtedly, Weibull distribution is used mostly in the area of reliability. This directly comes from its parametric character and wide possibilities to interpret very different laws of random variables by selection of corresponding values of parameters.

MATHEMATICAL MODEL OF WEIBULL DISTRIBUTION

Depending on number of parameters, there are the two- and three-parameter Weibull distributions. Expression for the survival function for a three-parameter model is [3, 4]:

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}, \; t \geq \gamma, \; \gamma \geq 0, \; \eta > 0, \; \beta > 0, \quad (1)$$

where:

t – is independent variable (time),
γ - is location parameter (parameter of minimal operation until failure),
η - is scale parameter and
β - is shape parameter.

If location parameter is $\gamma = 0$, the two-parameter Weibull distribution is obtained. Figure 1 presents charts of failure intensity function and density of operation time until failure for the two-parameter Weibull distribution and different values of shape parameter β [5, 6].

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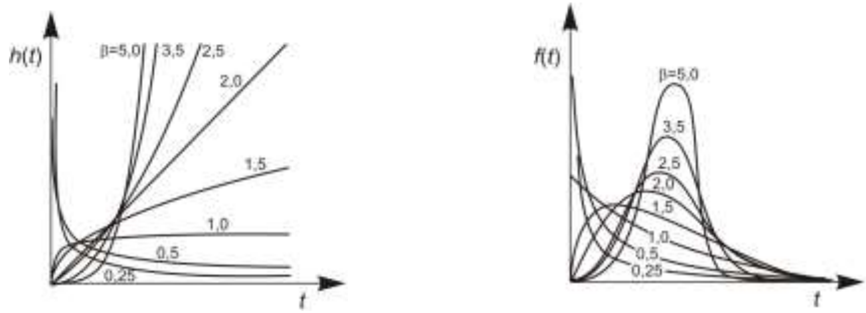
As it may be seen in the chart of failure intensity, *h*(*t*), for different values of the shape parameter, β, this distribution may be used as approximate model for all three periods of exploitation of machine systems' elements [5, 6]. In addition, if $\beta < 1$, the two-parameter Weibull distribution corresponds to hyper-exponential distribution. For $\beta = 1$, exponential distribution is gained, while for $\beta = 2$, Rayleigh distribution is gained. For greater values of parameter, β, Weibull distribution gets closer to normal distribution. For values of β between 3 and 4, differences between these two distributions are negligible. Nevertheless, it should be noted that theoretical laws are never mathematically identical.

Considering the previous and based on the results of approximation of empirical distribution with Weibull distribution, a model of other hypothetical distribution is determined more closely.



a) *b)*

Figure 1. Charts of functions of a) failure intensity and b) density of operation time until failure for the two-parameter Weibull distribution

In order to approximate the empirical distribution with the two- or three-parameter Weibull distributions, it is necessary to linearize mathematical models of distributions. With the three-parameter model, starting from expression (1), elementary mathematical transformations may change the expression $R(t) = 1 - F(t)$ into [4, 5, 7]:

$$\ln\ln\frac{1}{1-F(t)} = \beta \cdot \ln(t-\gamma) - \beta \cdot \ln\eta \,. \tag{2}$$

If the following substitutions are introduced:

$$y = \ln\ln\frac{1}{1-F(t)}, \tag{3}$$

$$a_1 = \beta, \tag{4}$$

$$x = \ln(t-\gamma) \text{ and} \tag{5}$$

$$a_0 = -\beta \cdot \ln\eta\,, \tag{6}$$

equation of the shape line is obtained:

$$y = a_0 + a_1 \cdot x \,. \tag{7}$$

A straight line in a coordinate system where *x*-axis has logarithmic and *y*-axis has double logarithmic scale may represent expression for reliability function of the three-parameter Weibull distribution. Linearization of reliability function of the two-parameter Weibull distribution is done in similar way. The only difference is that, instead of substitute $x = \ln(t-\gamma)$, a substitute $x = \ln t$ is used during transformation of coordinates.

DETERMINATION OF PARAMETERS OF THE TWO-PARAMETER WEIBULL DISTRIBUTION

Determination of parameters of the two-parameter Weibull distribution may be achieved graphically or analytically.

Goode and Kao had developed a **graphical procedure** for determination of Weibull distribution by the aid of probability paper [8]. In the two-parameter Weibull model, procedure demands that points with transformed coordinates (x_i, y_i), that is with coordinates $\left(\ln t_i, \ln\ln\left\{1/\left[1-F(t_i)\right]\right\}\right)$ are imported on Weibull probability paper (coordinate system with logarithmic scale on x-axis and double logarithmic scale on *y*-axis) [5]. If the arrangement of the points is approximately linear, the two-parameter Weibull distribution may be used for approximation. Procedure continues with approximation of a series of plotted points with a straight line. This approximation may be conducted graphically (by subjective assessment) or analytically (by the least squares method). If approximation is conducted with the least squares method, further course of graphical procedure has no meaning. Namely, based on determined coefficients a_0 and a_1 of regression line of the series, distribution parameters are easily achieved from expressions (4) and (6).

For graphical determination of scale parameter, η, the following condition is used:

$$t=\eta \quad \Rightarrow \quad F(t)=1-R(t)=1-\exp(-1)=0.632\,. \tag{8}$$

Based on this, parameter η is equal to *x*-coordinate of a point on approximate straight line that has *y*-coordinate equal to 0.632.

According to linearized model of Weibull distribution, shape parameter, β, represents inclination of approximate straight line in regard to *x*-axis. It is determined by drawing a line through point C on probability paper for Weibull distribution that is displayed within example in Figure 4 that is parallel to approximate line until it crosses a vertical line passing through point $x=-1$. From intersection point, a horizontal line is drawn until it crosses an axis for transformed *y*-coordinate. Value of parameter β is read directly on this axis with a plus sign.

During **analytical determination** of parameters of the two-parameter Weibull distribution using the least squares method, series of points with coordinates (x_i, y_i) is approximated by a straight line. The best of all approximate straight lines in a form (7) is the one for which the sum of squares of vertical offsets of the points from regression line is the smallest. Determination of coefficients a_0 and a_1 with the least squares method is done by previous determination of the expression for a sum of squares of ordinates offsets:

$$S(a_0,a_1)=(a_0+a_1x_1-y_1)^2+(a_0+a_1x_2-y_2)^2+\ldots$$
$$+(a_0+a_1x_n-y_n)^2=\sum_{i=1}^{n}(a_0+a_1x_i-y_i)^2. \tag{9}$$

Necessary and sufficient condition for function $S(a_0, a_1)$ to achieve the maximum is expressed by equation:

$$\frac{\partial S}{\partial a_0} = 2 \cdot \sum_{i=1}^{n} \left(a_0 + a_1 x_i - y_i \right) = 0 \,,$$
$$\frac{\partial S}{\partial a_1} = 2 \cdot \sum_{i=1}^{n} x_i \left(a_0 + a_1 x_i - y_i \right) = 0 \,, \tag{10}$$

from where a system of linear algebraic equations for determination of coefficients a_0 and a_1 is gained:

$$a_0 \cdot n + a_1 \cdot \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \,,$$
$$a_0 \cdot \sum_{i=1}^{n} x_i + a_1 \cdot \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i \,. \tag{11}$$

By determination of determinants of equation systems (11):

$$D = \begin{vmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{vmatrix}, \quad D_0 = \begin{vmatrix} \sum y_i & \sum x_i \\ \sum x_i y_i & \sum x_i^2 \end{vmatrix}, \quad D_1 = \begin{vmatrix} n & \sum y_i \\ \sum x_i & \sum x_i y_i \end{vmatrix}, \tag{12}$$

values of required coefficients are gained:

$$a_0 = \frac{D_0}{D} = \frac{\sum x_i^2 \cdot \sum y_i - \sum x_i \cdot \sum x_i y_i}{n \cdot \sum x_i^2 - \left(\sum x_i\right)^2},$$
$$a_1 = \frac{D_1}{D} = \frac{n \cdot \sum x_i y_i - \sum x_i \sum y_i}{n \cdot \sum x_i^2 - \left(\sum x_i\right)^2}. \tag{13}$$

Based on relationship between distribution parameters β, η and the coefficients of straight line equation, given by expressions (4) and (6), the following is obtained:

$$\beta = a_1 \text{ and } \eta = e^{-\frac{a_0}{a_1}} \,. \tag{14}$$

DETERMINATION OF PARAMETERS OF THE THREE-PARAMETER WEIBULL DISTRIBUTION

Procedures for determination of parameters of the three-parameter Weibull distribution are essentially different from previously described procedures. If operation time until the first failure occurs of the objects in the observed sample is not small in regard to total operation time, or if when entering the points onto probability paper, their layout is such that may be approximated by quadratic parabola convex upwards (in positive direction of y-axis), in that case, location parameter, γ, should be used. Value of this parameter is in the interval 0 to t_1, where t_1 is operation time until the first failure occurs on objects for a small sample or lower limit of the first interval for a large sample.

Value of location parameter, γ, of the three-parameter Weibull distribution may be determined graphically, graphically-analytically and analytically.

During **graphical determination** of location parameter, γ, after points with coordinates $\left(\ln t_i, \ln\ln\left\{1/\left[1-F(t_i)\right]\right\}\right)$ are entered on Weibull probability paper and conclusion that it is the three-parameter distribution is made, value of $\gamma = 0.9 \cdot t_1$ is taken in the first step of the iterative procedure [5, 7]. Points are entered onto probability paper again, now with coordinates $\left(\ln(t_i - \gamma), \ln\ln\left\{1/\left[1-F(t_i)\right]\right\}\right)$, and distribution of points is analysed. Feedback information for further steps of the procedure is gained based on distribution of points and possibility for approximation with a straight or a curved line, concave or convex upwards. Generally, three possibilities may occur in iterative procedure:

- if approximation curve obtained is still convex upwards, γ should be increased,
- if a straight line is obtained, γ is determined and procedure continues with determination of the rest of distribution parameters, and finally,
- if concave curve is obtained, γ should be decreased.

Iterative procedure continues until value of parameter γ is obtained, for which the points are located on a straight line. Afterwards, scale parameter, η, and shape parameter, β, are determined identically as for the two-parameter Weibull distribution.

In order to determine position parameter, γ, faster, **graphical-analytical** procedure has been developed. After entering the points onto probability paper and subjective approximation by a curved line, position parameter may be determined from the following expression [9]:

$$\gamma = t_2 - \frac{(t_3 - t_2)\cdot(t_2 - t_1)}{(t_3 - t_2) - (t_2 - t_1)}, \tag{15}$$

where:

t_1 – is *x*-coordinate of the first point of the curve (t_{min}),

t_2 – is *x*-coordinate of the point on the curve, which *y*-coordinate is arithmetic mean of *y* coordinates of points having *x* coordinates t_1 and t_3,

t_3 – is *x*-coordinate of the last point of the curve (t_{max}).

Value of t_2 is determined graphically. After determination of position parameter, γ, points with coordinates $\left(\ln(t_i - \gamma), \ln\ln\{1/[1 - F(t_i)]\}\right)$ are entered onto probability paper. Correctness of graphical-analytical procedure should be confirmed with the fact that points follow closely a straight line. After a best-fit straight line for a series of points is drawn, parameters η and β are determined in the same way as in the two-parameter Weibull distribution.

Analytical computer determination represents the third possibility for determination of parameter γ.

One of the possibilities for analytical computer determination of parameter γ is to approximate a series of points having transformed coordinates with a straight line. The idea is to approximate the points with the second order polynomial by the least squares method:

$$y = a_0 + a_1 x + a_2 x^2, \tag{16}$$

that is, by square parabola. Approximation of a series of points by the least squares method with square parabola is conducted in a similar way as approximation with a straight line. Firstly, an expression for a sum of squares of offsets is formed:

$$\begin{aligned} S(a_0, a_1, a_2) = {} & \left(a_0 + a_1 x_1 + a_2 x_1^2 - y_1\right)^2 + \\ & + \left(a_0 + a_1 x_2 + a_2 x_2^2 - y_2\right)^2 + \ldots \\ & + \left(a_0 + a_1 x_n + a_2 x_n^2 - y_n\right)^2 = \\ & = \sum_{i=1}^{n} \left(a_0 + a_1 x_i + a_2 x_i^2 - y_i\right)^2. \end{aligned} \tag{17}$$

By partial differentiation of expression (17) by coefficients a_0, a_1 and a_2, the system of linear algebraic equations for determination of these coefficients is obtained:

$$a_0 \cdot n + a_1 \cdot \sum_{i=1}^{n} x_i + a_2 \cdot \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i \,,$$

$$a_0 \cdot \sum_{i=1}^{n} x_i + a_1 \cdot \sum_{i=1}^{n} x_i^2 + a_2 \cdot \sum_{i=1}^{n} x_i^3 = \sum_{i=1}^{n} x_i y_i \,, \tag{18}$$

$$a_0 \cdot \sum_{i=1}^{n} x_i^2 + a_1 \cdot \sum_{i=1}^{n} x_i^3 + a_2 \cdot \sum_{i=1}^{n} x_i^4 = \sum_{i=1}^{n} x_i^2 y_i \,.$$

By determination of determinants of the system of equations (18):

$$D = \begin{vmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{vmatrix} \qquad D_0 = \begin{vmatrix} \sum y_i & \sum x_i & \sum x_i^2 \\ \sum x_i y_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 y_i & \sum x_i^3 & \sum x_i^4 \end{vmatrix},$$

$$D_1 = \begin{vmatrix} n & \sum y_i & \sum x_i^2 \\ \sum x_i & \sum x_i y_i & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^2 y_i & \sum x_i^4 \end{vmatrix}, \qquad D_2 = \begin{vmatrix} n & \sum x_i & \sum y_i \\ \sum x_i & \sum x_i^2 & \sum x_i y_i \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^2 y_i \end{vmatrix}, \tag{19}$$

values of required coefficients are obtained: $a_0 = \dfrac{D_0}{D}$, $a_1 = \dfrac{D_1}{D}$ i $a_2 = \dfrac{D_2}{D}$.

Based on curvature of the obtained squared parabola, that is on sign of coefficient a_2, it may be concluded whether the approximate curve is convex or concave in a positive direction of vertical curve. In this manner, similarly as in graphical procedure, feedback information is gained for further steps of the procedure. By searching through interval $0 \div t_1$, value γ is obtained for which the arrangement of points is closest to the straight line. For analytical determination of parameters of the three-parameter Weibull distribution with help of computer, a procedure shown by algorithm in Figure 2 is used.

According to this algorithm, a part of the computer program is formed for modelling the reliability. Searching through intervals of possible values of γ may be done in different ways. The algorithm solves this problem by halving the interval $0 \div t_1$ and investigating the curvature of the curve for $\gamma = t_1/2$. Based on a sign of second derivative of approximate polynomial (coefficient a_2), it may be concluded whether the value γ is smaller or larger than the used value. For $a_2 < 0$, γ takes value of a middle of the right interval ($\gamma = 0.75 \cdot t_1$), and for $a_2 > 0$, γ takes value of a middle of the left interval ($\gamma = 0.25 \cdot t_1$). Procedure continues until the width of the interval, where value of γ stands, becomes less than some value given in advance (e.g. ε = 0.001). If during iterative procedure, value $a_2 = 0$ occurs, this means that a straight line is obtained during approximation of a series of points, that is value of parameter γ is determined. If exiting from a cycle is a consequence of reduction of interval's width, a series of points is approximated by a straight line in order to determine coefficients of regression line. After the coefficients a_0 and a_1 are determined, the coefficients β and η are gained from the expression (14).

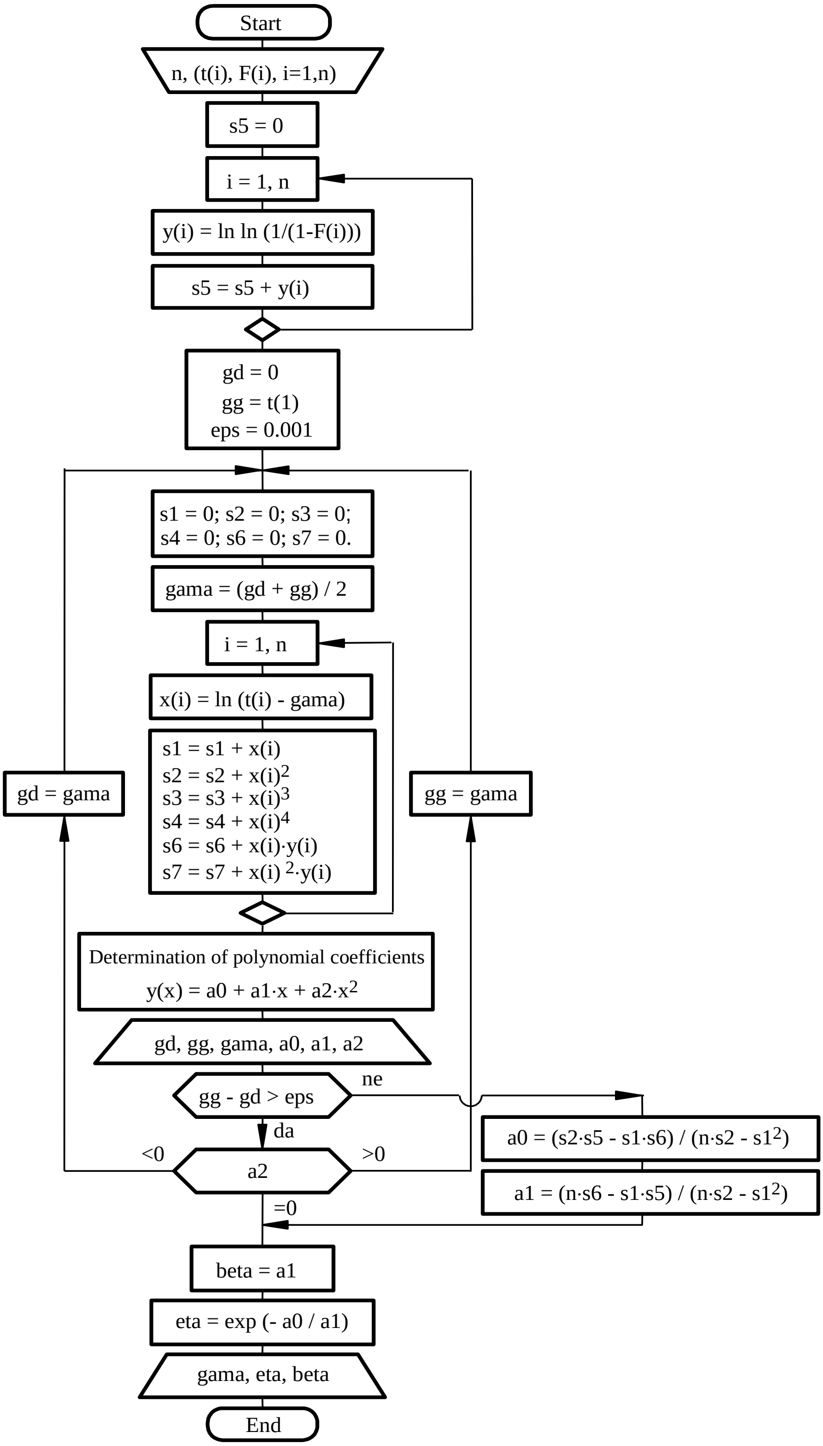


Figure 2. Algorithm of a program for determination of parameters of the three-parameter Weibull distribution

Graphical, graphical-analytical and analytical determination of parameters of the three-parameter Weibull distribution will be illustrated through the example of determining the distribution of operation time until failure of wheel's drum in the brake system of light commercial vehicles.

EXAMPLE

Sample of 65 identical drums in braking system of light commercial vehicles is tested for reliability assessment. Number of 7 intervals for grouping the values of random variable is adopted, based on expression $z = 1 + 3.3\log n$. Calculated width of the interval is Δt = 40,000 km, based on maximal and minimal values of random variable. Obtained values of operation time until failure of wheel's drum are grouped in time intervals and shown in Table 1. Parameters of Weibull distribution used as theoretical model of operation time until failure of wheel's drum should be determined.

Table 1. Number of failures of the wheel's drum in braking system by time intervals

Distance travelled x [10^3 km]	110÷150	150÷190	190÷230	230÷270	270÷310	310÷350	350÷390
Number of failures	9	13	17	11	8	5	2

With application of software for determination of theoretical model of empirical distribution, described in detail in [10], numerical characteristics of statistical series are gained:

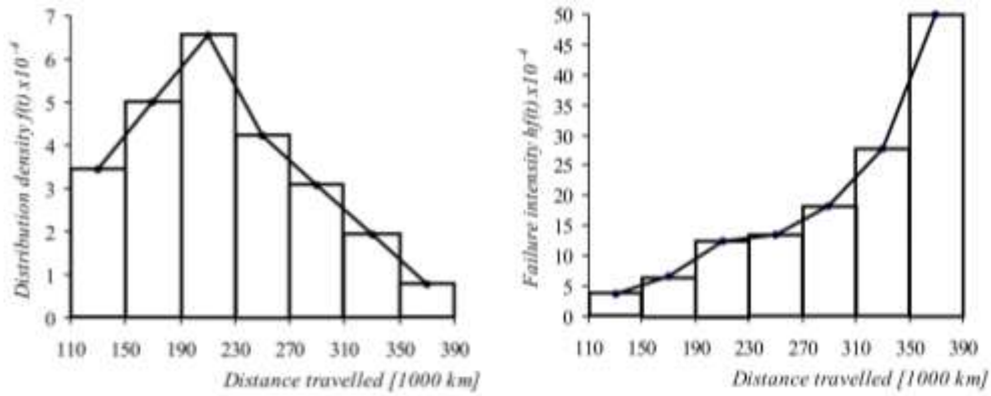
- mean value t_{sr} = 221,692;
- standard deviation σ = 63,429;
- median t_{50} = 214,706;
- mode M_o = 206,000;
- coefficient of asymmetry K_a = 0.417 and
- coefficient of flatness K_e = 2.439.

In continuation of a program, based on the procedures for assessment of functional indicators of the distribution of the random variable for a large sample (n> 30), estimated values of the number of correct objects $n(t)$, the reliability $R(t)$, the unreliability $F(t)$, density of operation time until failure $f(t)$ and failure intensity $h(t)$ of wheel's drum are gained for middles of time intervals and given in Table 2.

Table 2: *Estimated values of functional indicators of the distribution of the random variable*

i	*m(i)*	t_i	$n(t_i)$	$R(t_i)$	$F(t_i)$	$f(t_i)$	$h(t_i)$
1	9	130.00	60.5	0.93077	0.06923	0.34615E-02	0.37190E-02
2	13	170.00	49.5	0.76154	0.23846	0.50000E-02	0.65657E-02
3	17	210.00	34.5	0.53077	0.46923	0.65385E-02	0.12319E-01
4	11	250.00	20.5	0.31538	0.68462	0.42308E-02	0.13415E-01
5	8	290.00	11.0	0.16923	0.83077	0.30769E-02	0.18182E-01
6	5	330.00	4.5	0.06923	0.93077	0.19231E-02	0.27778E-01
7	2	370.00	1.0	0.01538	0.98462	0.76923E-03	0.50000E-01

Illustrations of graph charts of estimated values of density of operation time until failure, *f*(*t*), and failure intensity, *h*(*t*), wheel's drum, in the form of polygons and histograms, are given in Figure 3. In rough assessments, theses graph charts may serve for determination of hypothetical distribution models.



a) *b)*

Figure 3. Graphical display of estimated values of: a) density and b) intensity of wheel's drum failure

• Graphical determination of parameters of Weibull distribution using probability paper.

Usually, at the first step of graphical solving of task, approximation of a series of points is done by the two-parameter Weibull distribution. By transformation of coordinates and using the expressions $x_i = \ln t_i$ and $y_i = \ln\ln\{1/[1 - F(t_i)]\}$ and by entering the points onto probability paper for Weibull distribution (Figure 4), arrangement of points is gained that may be approximated by curve 1. Since the approximate function is convex, it means that the location parameter, γ, is positive. In the second step, series of points are approximated by the three-parameter Weibull distribution, with $\gamma = 0.9 \cdot t_1 = 99{,}000$ km. In this case, time, t_1 is a lower limit of the first interval. By repeated calculation of x_i coordinates according to expression $x_i = \ln(t_i - \gamma)$ and by entering the points onto probability paper, arrangement of points is obtained that may be approximated by curve 2. Since obtained curve is concave, γ should be smaller. Iterative procedure continues with decreasing values for γ with step $0.1 \cdot t_1$. Thus, for $\gamma = 0.7 \cdot t_1 = 77{,}000$ km approximately linear arrangement of point on probability paper is obtained. Parameter η value is equal to *x*-coordinate of the point on approximate straight line having *y*-coordinate 0.632, that is η = 163,000 km. Shape parameter, β, is determined as cathetus of right-angled triangle, whose other cathetus is equal to 1. Parameter value is read on auxiliary vertical axis for transformed coordinate *y*. In this particular case, value β = 2.38 is obtained.

• Graphical-analytical determination of location parameter γ.

In order to determine value of location parameter γ, according to expression (15), it is necessary to graphically determine the value for t_2. According to definition, t_2 is a value of *x*-coordinate of a point on approximate curve 1, whose *y*-coordinate is equal to arithmetic mean of *y*-coordinates of points with *x*-coordinates equal to t_1 and t_3. Thus, according to

Figure 4, orientation value of t_2 = 200,000 km is obtained. By using the expression (15), value of location parameter γ = 81,000 km, is obtained, which is nearly equal to the value obtained during graphical problem solving. By transformation of coordinates for calculated value for γ, arrangement of points that may be approximated by a straight line is obtained.

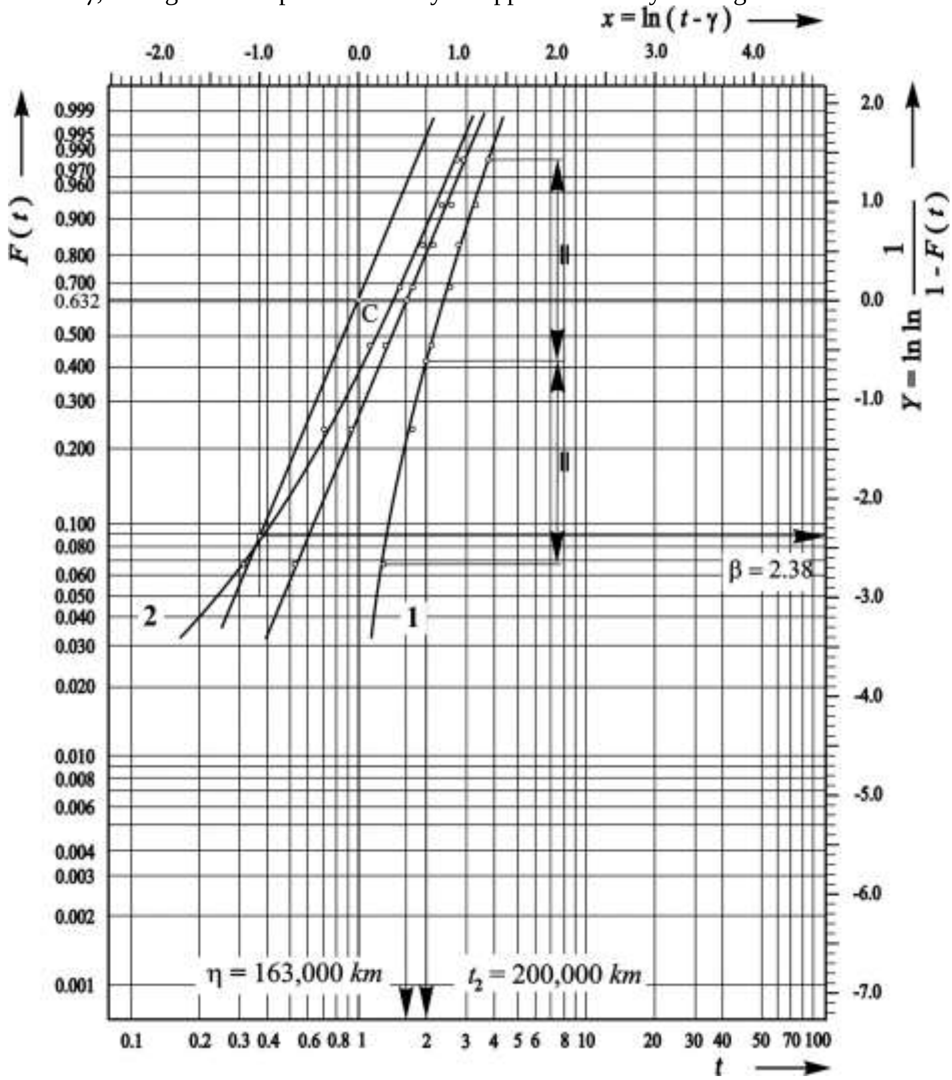


Figure 4. Determination of parameters of the three-parameter Weibull distribution using probability paper

• Analytical computer determination of parameters of Weibull distribution using the least squares method.

By approximation of empirical distribution of operation time until failure of wheel's drum by the three-parameter Weibull distribution and by using a computer program whose algorithm is presented in Figure 3, after 18 iterations by halving the intervals and determination of a sign of second derivative a_2, the parameters of the distribution are obtained: location parameter, γ = 76,115 km, scale parameter, η = 164,161 km and shape parameter, β = 2.355.

Based on this, the expression for probability of faultless operation of wheel's drum is:

$$R(t)=e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}=e^{-\left(\frac{t-76{,}115}{164{,}161}\right)^{2.355}}. \tag{20}$$

In order to determine validity of approximation, graphical testing over probability paper for Weibul distribution and nonparametric testing were conducted using tests of Kolmogorov, Pearson and Romanovsky. Table 3 presents values of transformed *x* and *y* coordinated for particular value γ = 76,115 km and Figure 5 presents arrangement of points on probability paper for Weibull distribution.

Table 3. Values of transformed coordinates

No.	t_i	$F(t_i)$	$x(i)$	$y(i)$
1	130.0	0.0692	3.987	-2.635
2	170.0	0.2385	4.542	-1.300
3	210.0	0.4692	4.897	-0.457
4	250.0	0.6846	5.158	0.143
5	290.0	0.8308	5.365	0.575
6	330.0	0.9308	5.537	0.982
7	370.0	0.9846	5.683	1.429

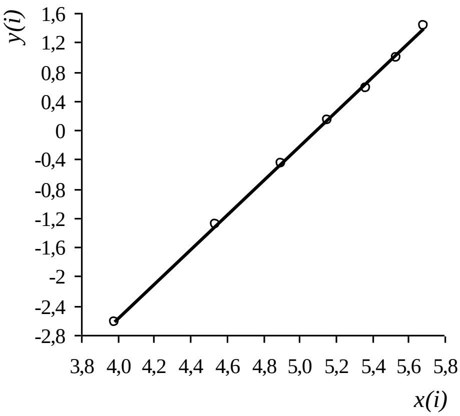


Figure 5. Arrangement of points on probability paper for Weibull distribution

Linear arrangement of the points in Figure 5 suggests that the approximate model satisfies conditions of graphical testing.

For testing of hypothetical distribution model, according to Kolmogorov test, it is necessary to determine the greatest absolute value of difference between theoretical model and estimated values of distribution functions of operation time until failure. Table 4 contains a segment of output list of a program that relates to this part. Figure 6 presents graphical representation of deviations of theoretical approximate model, $F_t(t)$, from empirical distribution, $F_e(t)$.

As it may be seen from Table 4, the largest deviation of theoretical model from empirical distribution is for the result No. 5 and amounts to 0.0143. For number of samples, *n* = 65 and given level of significance for Kolmogorov's test, α = 0.20, $\lambda_\alpha = 1.07$, permitted value of difference is:

$$D_n = \frac{\lambda_\alpha}{\sqrt{n}} = \frac{1.07}{\sqrt{65}} = 0.1327.$$

Since the maximal deviation is less than permitted value of difference, Weibull approximate distribution satisfies the Kolmogorov's test for adopted level of significance.

By application of Pearson's test procedure, value of $\chi^2 = 0.9711$ as a measure of deviation between empirical and approximate distribution is obtained. For number of intervals $z = 7$ and number of distribution parameters $l = 3$, number of degrees of freedom equals:

$$k = z - l - 1 = 7 - 3 - 1 = 3.$$

Table 4. Deviations of Weibull approximate curve from estimated values of distribution function of operation time until failure

No.	t_i	$F_e(t_i)$	$F_t(t_i)$	delta
1	130.0	0.0692	0.0700	0.0007
2	170.0	0.2385	0.2352	0.0032
3	210.0	0.4692	0.4613	0.0079
4	250.0	0.6846	0.6818	0.0028
5	290.0	0.8308	0.8451	0.0143
6	330.0	0.9308	0.9387	0.0080
7	370.0	0.9846	0.9806	0.0040

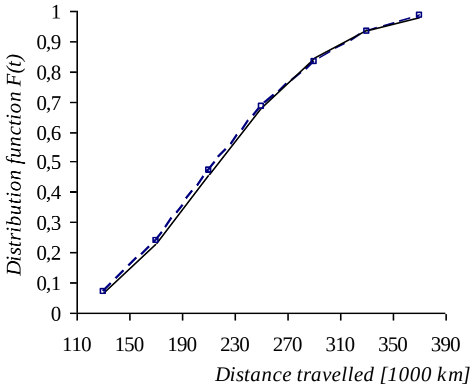


Figure 6. Graphical representation of deviations of Weibull approximate distribution from empirical distribution

By application of Pearson's test procedure, value of $\chi^2 = 0.9711$ as a measure of deviation between empirical and approximate distribution is obtained. For number of intervals $z = 7$ and number of distribution parameters $l = 3$, number of degrees of freedom equals:

$$k = z - l - 1 = 7 - 3 - 1 = 3.$$

Based on calculated values for χ^2 and number of degrees of freedom, *k*, and by looking at the table for χ^2 distribution, it may be concluded that Weibull distribution may be accepted as approximate model at level of significance $\alpha = 0.80$.

Comparable value for Romanovsky's test is:

$$R_o = \frac{\left|\chi^2 - k\right|}{\sqrt{2k}} = \frac{\left|0.9711 - 3\right|}{\sqrt{2 \cdot 3}} = 0.828,$$

which is smaller than 3 and it means that Weibull distribution meets criterion of Romanovsky's test.

CONCLUSIONS

Procedure for graphical determination of location parameter, γ, of the three-parameter Weibull distribution is long lasting and liable to errors due to imprecise entering of points onto probability paper, subjectivity during assessment of drawing of approximate straight lines and curves and impossibility to precisely read the parameter value.

Since graphic-analytic procedure is largely based on graphical representation of points on probability paper and corresponding approximations, everything that has been said on graphical method applies also to graphical analytical method.

Program determination of parameters of the three-parameter Weibull distribution enables gaining desired accuracy of the results with great speed. Thanks to that and known features of this distribution regarding possibility of approximation of empirical distribution of random variables, in great number of cases, Weibull distribution is optimal solution, with respect to other theoretical models.

Based on the graphic of the estimated values of failure intensity of wheel's drum in braking system of light commercial vehicles, it can be concluded that these are failure modes that occur during object aging. Theoretical approximate model of Weibull distribution satisfies graphics tests and analytical nonparametric tests with a high level of significance.

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