ASSESSMENT OF THE OPTIMIZATION PROCEDURE FOR THE NONLINEAR SUSPENSION SYSTEM OF A HEAVY VEHICLE

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1. INTRODUCTION

Suspension systems influence the overall performance of the vehicle by receiving the loads created by the road excitation. These loads are transmitted through the tires and the wheels ensuring that vibrations are isolated and not perceived by the passengers. Depending on the aspect of interest regarding the design of a suspension system, the focus of the studies is turned on the ride comfort or the road holding of the vehicle being the basic needs for a good suspension system. Ride comfort is related to the passenger's perception of the moving vehicle's environment, while road holding is the degree to which a car maintains contact with the road surface in various types of directional changes. Keeping the tires in contact with the ground constantly is of vital importance for the friction between the vehicle and the road affecting the vehicle's ability to steer, brake and accelerate. Time domain statistics, such as mean suspension deflection, maximum and RMS values of suspension acceleration are often used in suspension design as criteria for road comfort ability. The main conflict and common trade-off in the automotive industry is the one concerning the displacement and the acceleration of the suspension. A hard configuration with high spring stiffness and high damping is required for reducing the suspension displacement. On the other hand, low spring stiffness and low damping is required for reducing suspension acceleration. This conflict depicts the trade-off between the ride comfort and the road holding.

Multibody dynamics have been used extensively by automotive industry to model and design vehicle suspension. Before modern optimization methods were introduced, design engineers used to follow the iterative approach of testing various input parameters for vehicle suspension performance, setting as targets predefined performance indexes so as to be achieved. With the advent of various optimization methods along with developments in computational studies, the design process has been speeded up to reach to optimal values of the design parameters. Many studies, turned their attention to the optimization of the suspension systems, so as to facilitate the influence of design parameters in order to get the minimum or the maximum of an objective function subjected to certain constraints. These constraints depicted the practical considerations into the design process.

The issue of the most appropriate objective function is the main subject of intense studies in order to be able to combine many aspects of the dynamical behaviour and overcome the aforementioned conflict of ride comfort and road holding. Georgiou et al [1].

Used a sum of the variances of the body acceleration, the suspension travel and the one of the tire forces as a fitness function. On the other hand, Ozcan et al. [2] used as fitness

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function the sum of the RMS value of the weighted body acceleration and the difference between the maximum and the minimum force applied to the tire. While Shirahatt et al. [3] selected the root mean square of the passenger's acceleration. Another index of performance was proposed by Gündoğdu et al [4], examining also the effect that the vehicle has on the passenger's body by adding to the fitness function, terms concerning the variance of the head's acceleration and the crest factor. Another approach regarding the fitness function included the use of the constraints as terms in the fitness function with suitable weights [5], depending on the importance of each constraint or as separate functions to the optimization algorithm [3]. This approach was followed also by Koulocheris et al. [6], where the maximum value of vertical acceleration of the vehicle body at the passenger seat was minimized from the view point of ride comfort adding a quadratic penalty of the sum of constraints functions.

In this paper the optimization of a suspension system is studied. More specifically, not only the efficiency of different methods is investigated but also the efficiency of various fitness and objective functions. Three optimization methods were used: Genetic Algorithms, Gradient Based and a hybridization of the above algorithms. In conclusion this paper is organized as follows: in Section 1 the model used for the optimization of the heavy vehicle is described as well as the road excitation applied, in Section 2 the methods are presented while in Section 3, the optimization procedure applied in the problem is analysed, in Section 4 and 5 the results of the current study are illustrated and discussed, and finally in Section 5 conclusions and future work are displayed.

2. VEHICLE MODEL

In this paper, a heavy vehicle was modelled as a Half Car Model, as shown in figure 1, so as to examine the vertical vibrations that are induced from the road. This model simulates the front and rear axle of the vehicle and it allows the pitch phenomena to be observed.

2.1 Equations of motion

The mass of the body of the vehicle is considered as a rigid bar. The body of the vehicle has a mass m_s, which is half of the total body mass, and lateral moment of inertia I_z, which is the half of the total body mass moment of inertia. The unspring masses for the front and rear wheel are m_F and m_R, respectively. The distance of the front and rear axle from the center of mass are a_F and a_R, respectively (Figure 1). Moreover, the tires are modelled with a spring system indicated by different parameters for the front and rear tires K_{T_F} and K_{T_R}, respectively and were evaluated experimentally on previous work [9]. The tires receive as input the road excitations, $z_{Road\ F}$ and $z_{Road\ R}$, which will be described in the next subchapter. The parameters of the vehicle are displayed in Tables 1 and 2.

A suspension system has to adjust to the irregularities of the road surface, in order to ensure the comfort of the passengers and the holding of the vehicle. This adjustment is related to certain nonlinearities in the main components of the suspension, which can be observed mostly in active suspension systems. But nonlinearities can be found in passive suspension systems too, with the addition of nonlinear terms in either the springs or the dampers of the suspension. In this work, the nonlinearity of the suspension spring is studied. The spring suspension force can be mathematically described as:

$$F_{spring} = K_l \cdot x + K_{nl} \cdot x^3 \quad (1)$$

Table 1 Parameters of Half Car Model

Values of Vehicle Parameters	
m_s=2220 *[kg]*	
I_z =1142 *[kg m^2]*	
m_F=50 *[kg]*	m_R=100 *[kg]*
a_F=1.61 *[m]*	a_R=1.67 *[m]*
K_{T_F}=4*10^5 *[N/m]*	$K_{T_R} = 2K_{T_F}$

Table 2 Nomenclature of Vehicle Parameters

Parameters		Subscirpts	
z	Vertical motion coordinate	S	Body
θ	Pitch motion coordinate	F	Front
z_{Road}	Road excitation	R	Rear
m	Mass	l	Linear
C	Damper's coefficient	nl	Non-linear
K	Spring's stiffness coefficient	T	Tire
a	Distance of the center of mass of the vehicle		

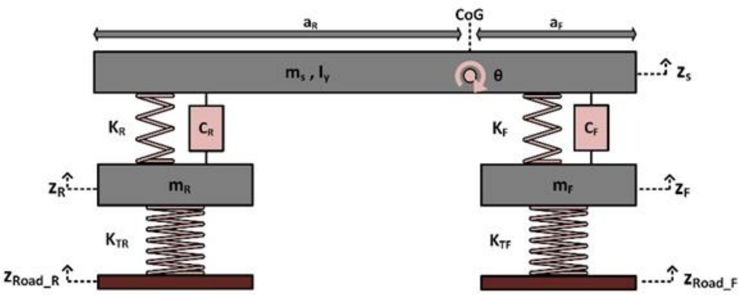


Figure 1 Half Car Model

The derivation of the equations is based on rigid body theory and on the assumption of small angles; sinθ and cosθ are approximated as θ and 1, respectively. Thus, the governing equations of the model are:

Body Bounce:

$$m_s \cdot \ddot{z}_s + C_F \cdot (\dot{z}_s - \dot{z}_F - a_F \cdot \dot{\theta}) + C_R \cdot (\dot{z}_s - \dot{z}_R + a_R \cdot \dot{\theta})$$

$$+K_{l_F} \cdot (z_s - z_F - a_F \cdot \theta) + K_{l_R} \cdot (z_s - z_R + a_R \cdot \theta) \tag{2}$$

$$- K_{nl_F} \cdot (z_s - z_F - a_F \cdot \theta)^3 + K_{nl_R} \cdot (z_s - z_R + a_R \cdot \theta)^3 = 0$$

Body Pitch:

$$I_z \cdot \ddot{\theta} - \alpha_F \cdot C_F \cdot \left(\dot{z}_s - \dot{z}_F - a_F \cdot \dot{\theta}\right) + a_2 \cdot C_R \cdot \left(\dot{z}_s - \dot{z}_R + a_R \cdot \dot{\theta}\right)$$
$$- \alpha_F \cdot K_{l_F} \cdot (z_s - z_F - a_F \cdot \theta) + \alpha_2 \cdot K_{l_R} \cdot (z_s - z_R + a_R \cdot \theta) \quad (3)$$
$$- \alpha_F \cdot K_{nl_F} \cdot (z_s - z - a_F \cdot \theta)^3 + \alpha_2 \cdot K_{nl_R} \cdot (z_s - z_R + a_R \cdot \theta)^3 = 0$$

Front Wheel Bounce:

$$m_F \cdot \ddot{x}_F - C_F \cdot \left(\dot{z} - \dot{z}_F - a_F \cdot \dot{\theta}\right) - K_{l_F} \cdot (z_s - z_F - a_F \cdot \theta)$$
$$-K_{nl_F} \cdot (z_s - z_F - a_F \cdot \theta)^3 + K_{T_F} \cdot \left(z_F - z_{Road_F}\right) = 0 \quad (4)$$

Rear Wheel Bounce:

$$m_R \cdot \ddot{x}_R - C_R \cdot \left(\dot{x}_s - \dot{x}_R + a_R \cdot \dot{\theta}\right) - K_{l_R} \cdot (x_s - x_R + a_R \cdot \theta)$$
$$-K_{nl_R} \cdot (z_s - z_R + a_R \cdot \theta)^3 + K_{T_R} \cdot \left(x_R - z_{Road\,R}\right) = 0 \quad (5)$$

One of the most important parameters of a vehicle model are the suspension travel as well as the tire deflection. The suspension travel of the suspension is the term $(z_s - z_R - a_R \cdot \theta)$ and $(z_s - z_R - a_R \cdot \theta)$ for the front and rear suspension respectively and the tire deflection is $\left(z_F - z_{Road_F}\right)$ and $\left(z_R - z_{Road_R}\right)$ for the front and rear tire respectively.

2.2 Road Excitation

Generally, the subject of road excitation is important, since it allows researchers to investigate realistic road profiles through simulation. In this study, a road bump was generated mathematically, with a half-sinusoid excitation function. The height of the bump was selected as h=0.05 m with an appropriate length for the half-sinusoid of L=2 m. The vehicle velocity was constant and at 10 m/s. As a function of time, the road conditions could be given by:

$$y_F = \begin{cases} h \cdot \sin(w \cdot t), & if\ t_0 \le t < t_0 + \dfrac{L}{2 \cdot V} \\ 0, & otherwise \end{cases} \quad (6)$$

$$y_R = \begin{cases} h \cdot \sin(w \cdot t), & if\ t_0 + t_{distance} \le t < t_0 + \dfrac{L}{2 \cdot V} + t_{distance} \\ 0, & otherwise \end{cases} \quad (7)$$

where t_0 is the starting time of the road bump, $t_{distance}$ is the time lag between front and rear wheels ($\frac{a_F + a_R}{V}$), whilst w is the excitation frequency $\frac{2 \cdot \pi \cdot L}{V}$. More specifically, the front and rear wheels follow the same trajectory with a time delay $t_{distance}$, which is due to the distance $a_F + a_R$ of front and rear wheels. The excitation is illustrated in Figure 2.

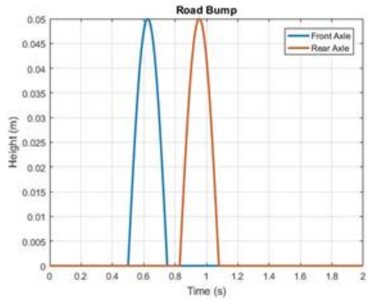


Figure 2 Road Bump based on sinusoidal function

3. OPTIMIZATION METHODS

In this paper, three optimization methods were examined. At first, Genetic (GA) and Gradient Based (GB) Algorithms were used in order to investigate their efficiency as well as to validate and understand their behaviour based on the theory of previous work [7 and 8]. Then, a hybridization of the above algorithms is tested in order to combine the advantages of each method and compare it with them.

Hybrid Algorithms form a new area of interest for the research community, and the optimization methods would not be the exception. Hybrid optimization algorithms combine two or more different optimization methods in order to solve a problem, switching between them over the course of the algorithm. In this way all the advantages of the involved methods are drafted in order to achieve the optimum result. For example, GA are more likely to find a global minimum, contrary to the GB which are often trapped, as well as the fact that the GA do not require the calculation of any derivative. Thus, the fitness function of the GA does not need to be continuous, so they are able to handle problems with discrete solution spaces. Furthermore, GB should be used when the area of the desired solution is known, in any other case the GA have better results due to their stochastic nature. Based on these points, a possible combination would be a stochastic method followed by a deterministic one. In the beginning. GA will operate for a number of generations with large population in order to locate the area of the optimum solution. After that, GB is employed so as to locate the global minimum, knowing the area of the desired solutions. The GA set of optimal values is used as initial value for the GB method, and the upper and lower bounds are set in a symmetric area around the initial values. Thus, the ability of GB methods to converge to a local (in this case global) minimum is exploited. The genetic part in the hybrid algorithm was active for 10 generations.

4. OPTIMIZATION PROCEDURE

The object of the optimization is the vehicle model, mentioned in chapter 1.1, and the target of the optimization is the optimum dynamic behaviour. The design variables selected are all the suspension parameters and specifically the vector below:

$$Design\ Variables = \left[K_{l_F}\ , K_{nl_F}, K_{l_R}, K_{nl_R}, C_F, C_R\right] \tag{8}$$

which are the linear and the nonlinear part of the spring as well as the damping coefficient of both the front and rear suspension system, see also Table 2.

Regarding the bounds of the design variables, the upper and lower ones were chosen based on experimental processes, previous works and the literature: They are presented in Table 3. As far as the constraints are concerned, they were chosen in terms of the dynamic behaviour of the vehicle and the design of the suspension. Specifically, the root mean square of body's acceleration as well as the contribution of the non-linear term of the spring force were selected as the constraints of the optimization process being set under 1 (m/s2) and between 10-30% respectively.

Table 3 Lower and Upper Bounds of the Design Variables

Design Variables	Lower Bounds	Upper Bounds
$\boldsymbol{K_{l_F}}$, $\boldsymbol{K_{l_R}}$ (N/m)	3.2·104	1.5·105
$\boldsymbol{C_F}$, $\boldsymbol{C_R}$ (N/m·s)	2.0·103	1.0·104
$\boldsymbol{K_{nl_F}}$, $\boldsymbol{K_{nl_R}}$ (N/m3)	5·105	3·108

While regarding the fitness functions, it was decided to investigate single-objective problems in comparison with multi-objective ones. Thus, three single-objective problems were formulated regarding different targets of the dynamic behavior of the vehicle. The first target is the ride comfort of the driver and the safety of the truck load. This target is evaluated through the vehicle's body acceleration (*$fitness_1$ - Case 1*). The second one is the travel of both front and rear suspensions (*$fitness_2$ - Case 2*). The third one is the deflection of both front and rear tires, ensuring road holding (*$fitness_3$ - Case 3*). The mathematical equations of these targets were formed to the following three fitness functions (equation 9-11). In order to achieve better results, the variances (will be used as var (x)) of the values, which depict the optimization targets, were selected. Moreover, so as to include in the terms $fitness_2$ and $fitness_3$ both the values of the front and rear suspension travel and the values of front and rear tire deflection respectively, the average value of their variances was used. To conclude, three single objective problems were built with the following fitness and objective functions:

$$fitness_1 = var(acc_{body}) \tag{9}$$

$$fitness_2 = \frac{1}{2} \cdot [var(suspension.travel_F) + var(suspension.travel_R)] \tag{10}$$

$$fitness_3 = \frac{1}{2} \cdot [var(tire.deflection_F) + var(tire.deflection_R)] \tag{11}$$

Moreover, due to the fact that the three targets selected are in conflict, the optimal solution for each target would be different. Taking this conflict into consideration, the aforementioned targets were combined so as to formulate a multi-objective problem. In this way, it was possible to examine the balancing between the desired goals of the optimization and locate the optimum solution of the problem. In an attempt to save computational time, reduce the complexity of the problem and concentrate more to the comparison of the optimization methods, the multi-objective problem was converted into a single objective one through the sum of the three targets, mentioned in equations 9-11, as follows:

$$fitness = w_1 \cdot fitness_1 + w_2 \cdot fitness_2 + w_3 \cdot fitness_3 = f_1 + f_2 + f_3 \tag{12}$$

where w_1, w_2, w_3 are the weight factors. In order to investigate the multi-objective approach more accurately, two scenarios were tested. In the first scenario the magnitudes of all the three terms (*f_1, f_2 and f_3*) were at the same order and balanced (*Case 4)*. While in the second scenario with different weight factors, the term f_1 was selected as the main one and its magnitude was set one order greater than the ones of the two other terms, f_2 and f_3 (*Case 5*). The values of the weight factors were selected based on random simulations of the model.

The optimization was implemented with the Optimization Toolbox of MATLAB R2016a, provided for academic use by NTUA. The methods selected for the configured problem was a Genetic Algorithm (*ga of MATLAB*), a Gradient Based Algorithm (*active set of fmincon of MATLAB)* as well as a hybridization of the above. To sum up, 5 different scenarios (*S1-S5*), regarding the optimization methods, were implemented for 5 different cases as far as the fitness functions are concerned (*Case 1-5*). The set of the optimization scenarios is illustrated in Table 4 in detail. In S1 and S3, the population size was set to 200 as proposed in MATLAB R2016a for problems with more than 5 design variables (currently 6), whilst in S2 and S4, the population size was set to 1000 in order to investigate the influence of the population size. As far as the hybrid method is concerned, the part of the genetic algorithm was active only for 10 generations and then the gradient based algorithm was enabled.

Table 4 Implemented Optimization Scenarios for each Case

Implemented Optimization Scenarios for each Case					
Genetic Algorithm	S1	*Population Size*	200	*Fitness Function Tolerance*	10^{-6}
	S2	*Population Size*	1000		
Hybrid Algorithm	S3	*Population Size*	200	*Fitness Function Tolerance*	10^{-6}
	S4	*Population Size*	1000		
Gradient Based	S5	*Objective Function Tolerance*		10^{-6}	

5. RESULTS

The results will be presented for each case of fitness and objective function (*Case 1-5*). At first, in each case the optimal design variables will be presented for every optimization scenario (S1-S5). Furthermore, the RMS of the vehicle's body acceleration, the maximum suspension travel of both front and rear suspensions and the maximum tire deflection through the front and rear tire forces, which are $K_{T_i}(x_i - y_i)$, will be illustrated after the simulation of the model for the optimal design variables.

5.1 Case 1

In this case, the fitness function was the variance of body's acceleration. In terms of the objective function, the "best" optimal solution was found with optimization scenarios S4 and S5, which are the hybrid one with population 1000 and the gradient based algorithm respectively. In these scenarios, the fitness function, hence the term that depicts the ride comfort, reached to the minimum value, as it is shown both in Table 6, through the RMS of vehicle's body acceleration, and in Figure 3, through the variance of vehicle's body acceleration and term f1. Moreover, as far as the design variables are concerned, these two scenarios have converged to design variables closed to each other's proving that the algorithms converged almost to the same solution. In addition, the contribution of the nonlinear part of the suspension spring is almost the same. In Table 5, the optimal solutions of the design variables verify the target of the optimization of this case, since in order to improve and secure the ride comfort, the optimization methods are trying to configure the suspension system with low spring stiffness and low damping coefficient sacrificing the suspension travel. Finally, the computational time needed for the optimal solutions has to be mentioned due to the fact that the gradient based algorithm found the optimal solution in the 5% of the time that the hybrid algorithm converged.

Table 5 Optimal Solutions of Design Variables / Case 1

Design Variables	Optimization Scenarios				
	S1	S2	S3	S4	S5
K_{l_F}(N/m)	102620	34479	105019	43094	36194
C_F(N·s/m)	6950	2000	5890	2050	2582
K_{l_R}(N/m)	32352	74939	58027	33652	39741
C_R(N·s/m)	2083	5112	2155	2272	3228
K_{nl_F}(N/m3)	1.97*107	3.42*106	4.08*107	1.09*107	1.03*107
K_{nl_R}(N/m3)	8.58*106	4.47*107	1.30*107	9.19*106	1.18*107

Table 6 Vehicle Model's Parameters for the Optimal Solutions / Case 1

Vehicle Model's Parameters	Optimization Scenarios				
	S1	S2	S3	S4	S5
RMS(accbody) (m/s^2)	0.631	0.561	0.682	0.450	0.470
Max. Suspension Travel Front (m)	0.025	0.037	0.024	0.040	0.037
Max. Suspension Travel Rear (m)	0.038	0.027	0.038	0.038	0.037
Max. Tire Force Front (N)	4085	2249	3833	2991	2649
Max. Tire Force Rear (N)	3061	3750	3275	2610	2791
Nonlinear% of Front Spring Force	11	12	19	29	28
Nonlinear% of Rear Spring Force	27	30	24	29	28

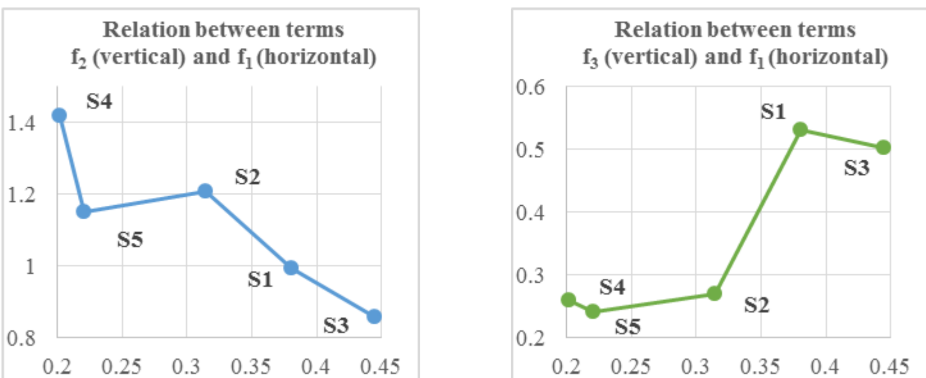


Figure 3 Relation between terms (a) f2 and f1 (b) f3 and f1 / Case 1

5.3 Case 2

In this case, the fitness function was the average of the variances of both front and rear suspension travels. In terms of the objective function, the "best" optimal solution was found with optimization scenarios S2, S3 and S4 which are the genetic algorithm with population 1000 and the hybrid ones with population 200 and 1000, respectively. In these scenarios, the fitness function, hence the term that depicts the suspension travel, reached to the minimum value of all the scenarios as it is shown both in Table 8, through the maximum value of both the front and rear suspension travels, and in Figure 4, through the average of the variances of the front and rear suspension travels, hence term f2. Moreover, as far as the design variables are concerned, the two scenarios, S2 and S4, have reached to close values proving that the algorithms converged to almost the same optimal solution.

Table 7 Optimal Solutions of Design Variables / Case 2

Design Variables	Optimization Scenarios				
	S1	S2	S3	S4	S5
K_{l_F}(N/m)	83283	51337	38952	57879	51377
C_F(N·s/m)	6635	9696	8003	8759	2003
K_{l_R}(N/m)	70210	36651	47540	36934	36456
C_R(N·s/m)	9091	8757	8286	9376	2001
K_{nl_F}(N/m^3)	$2.57*10^7$	$2.48*10^7$	$2.56*10^7$	$1.30*10^7$	$7.56*10^7$
K_{nl_R}(N/m^3)	$3.42*10^7$	$1.37*10^7$	$1.60*10^7$	$1.84*10^7$	$2.96*10^7$

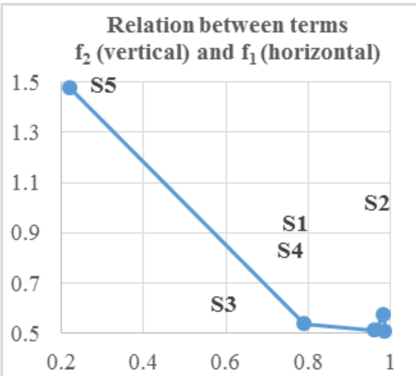
On the other hand, the optimization scenario S3 differs mainly on the linear part of the spring both to the front and the rear suspensions, proving that has selected a solution with different characteristics which could also be shown by its lower value of the RMS of the vehicle's body acceleration. In Table 7, the optimal design variables of the current case verify the target of the optimization, due to the configuration of both higher spring stiffness and damping coefficient in order to secure the minimum suspension travel in contrary to the previous case. In addition, the failure of the gradient based algorithm has to be mentioned, due to the lack of compability between both the design variables in Table 7 and the vehicle parameters in Table 8 with the objective of the optimization, which is the minimization of the suspension travel.

Table 8 Vehicle Model's Parameters for the Optimal Solutions / Case 2

Vehicle Model's Parameters	Optimization Scenarios				
	S1	S2	S3	S4	S5
RMS(accbody) (m/s^2)	0.989	0.994	0.908	0.980	0.470
Max. Suspension Travel Front (m)	0.030	0.025	0.026	0.026	0.041
Max. Suspension Travel Rear (m)	0.020	0.021	0.022	0.021	0.038
Max. Tire Force Front (N)	4682	4168	3571	3957	3212
Max. Tire Force Rear (N)	3559	3276	3384	3364	3462

Nonlinear % of Front Spring Force	22	23	30	13	20
Nonlinear % of Rear Spring Force	16	15	14	18	10

Finally, the computational time needed for the optimal solutions for optimization scenario S4 has dropped to the 50% of the time needed in S2, proving the importance of the hybrid algorithm and its effectiveness, as in both scenarios the optimal solution are similar. Additionally, in S3 due to the lower population and the hybrid algorithm, the problem converged to almost the 10% of the time in comparison with S2.



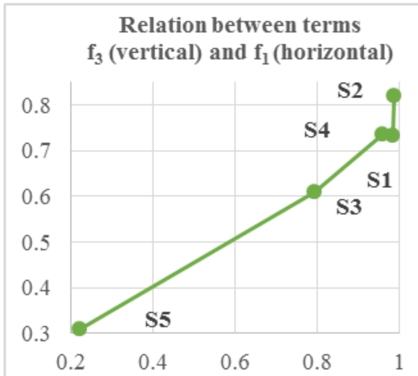


Figure 4 Relation between terms (a) f2 and f1 (b) f3 and f1 / Case 2

5.3 Case 3

In this case, the fitness function was the average of the variances of both front and rear tire deflections. In terms of the objective function, the "best" optimal solution was found by the optimization scenarios S2 and S3 which are the genetic algorithm with population 1000 and the hybrid one with population 200. In these scenarios, the fitness function, hence the term that depicts the tire deflection, reached to the minimum value of all the scenarios as it is shown both in Table 10, through the maximum value of both the front and rear tire forces, and in Figure 5, through the average of the variances of the front and rear tire deflections, hence term f_3. Moreover, as far as the design variables are concerned, the two scenarios, S2 and S3, have reached to close values proving that the algorithms found almost the same optimal solution. The only difference between the design variables in these two scenarios is the contribution of the nonlinear part of the spring of both front and rear suspensions. Despite the different population used in this two scenarios (*S2 and S3*), the hybrid algorithm with the lower population (*S3*) has succeeded in comparison with the genetic algorithm with the greater population (*S2*). The hybrid algorithm overcame the disadvantage of the lower population combining the advantages of both the genetic and the gradient based algorithm, leading to the optimal solution in a computational time of almost 8% of the time of optimization scenario S2. The success is confirmed due to the close solutions of the design variables as well as the values of the important parameters of the vehicle's model. Finally, the effectiveness of the gradient based algorithm, in this current case, is based on the connection of the tire deflection and the RMS of vehicle's body acceleration, which is illustrated in Figures 3b-5b of all the cases. Particularly, the increase or the decrease of the term f_1, which depicts the ride comfort, leads to the increase or the decrease of term f_3, which depicts the tire deflection, respectively.

Table 9 Optimal Solutions of Design Variables / Case 3

Design Variables	Optimization Scenarios				
	S1	S2	S3	S4	S5
K_{l_F}(N/m)	41279	33114	37028	47186	33583
C_F(N·s/m)	3632	2139	2092	2203	4242
K_{l_R}(N/m)	77718	65367	67512	57595	37473
C_R(N·s/m)	4319	3661	4211	2713	2000
K_{nl_F}(N/m^3)	1.44*10^7	4.95*10^6	1.11*10^7	8.54*10^6	1.23*10^7
K_{nl_R}(N/m^3)	5.00*10^7	2.69*10^7	3.47*10^7	2.62*10^7	3.31*10^6

Table 10 Vehicle Model's Parameters for the Optimal Solutions / Case 3

Vehicle Model's Parameters	Optimization Scenarios				
	S1	S2	S3	S4	S5
RMS(accbody) (m/s^2)	0.637	0.520	0.556	0.529	0.466
Max. Suspension Travel Front (m)	0.033	0.036	0.037	0.038	0.031
Max. Suspension Travel Rear (m)	0.025	0.032	0.029	0.031	0.040
Max. Tire Force Front (N)	2936	2226	2515	2853	2596
Max. Tire Force Rear (N)	3381	3537	3569	3006	3111
Nonlinear % of Front Spring Force	27	16	30	21	26
Nonlinear % of Rear Spring Force	29	29	30	30	12

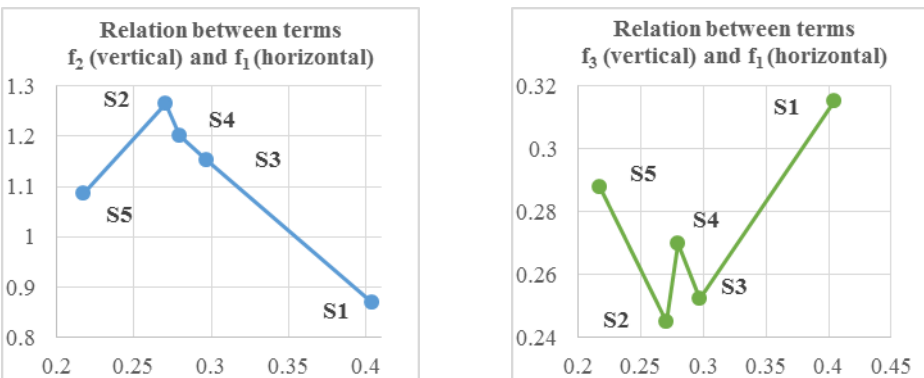


Figure 5 Relation between terms (a) f2 and f1 (b) f3 and f1 / Case 3

5.4 Case 4

In this case, the fitness function was the sum of all the three terms (f1, f2 and f3) which were at the same order and balanced. First of all, as far as the effectiveness of the algorithms is concerned, the gradient based wasn't able to find a "good" optimal solution, getting trapped probably in a local minimum. This conclusion is verified by the fact that all the design variables reached the lower bounds without giving a proper solution, while the optimization process ended in only 43 seconds. Due to the multi-objective character of this case it's difficult to understand which optimization scenarios converged to the "best" optimal solutions. On the other hand, the differences in the optimal design variables in every optimization scenario have to be mentioned. In contrary with the previous cases, none of the optimal design variables are close enough to indicate the same characteristics of the solutions, but Table 12, based on the values of important parameters of the vehicle model, points out some common characteristics between optimization scenarios of S2-S4, such as the maximum suspension travels and the maximum tire forces indicating probably common characteristics to the solutions.

Table 11 Optimal Solutions of Design Variables / Case 4

Design Variables	Optimization Scenarios				
	S1	S2	S3	S4	S5
K_{l_F}(N/m)	81307	46116	63116	42377	32000
C_F(N·s/m)	5431	4129	3645	4002	2000
K_{l_R}(N/m)	57264	45371	66731	60069	32000
C_R(N·s/m)	2877	5301	2753	3693	2000
K_{nl_F}(N/m^3)	$2.12*10^7$	$6.24*10^6$	$1.78*10^7$	$1.36*10^7$	$5.00*10^6$
K_{nl_R}(N/m^3)	$9.52*10^6$	$2.73*10^7$	$3.47*10^7$	$2.86*10^7$	$1.08*10^7$

Table 12 Vehicle Model's Parameters for the Optimal Solutions / Case 4

Vehicle Model's Parameters	Optimization Scenarios				
	S1	S2	S3	S4	S5
RMS(accbody) (m/s2)	0.666	0.611	0.635	0.589	0.370
Max. Suspension Travel Front (m)	0.028	0.033	0.031	0.031	0.039
Max. Suspension Travel Rear (m)	0.033	0.027	0.028	0.029	0.048
Max. Tire Force Front (N)	3704	3096	3270	2802	2103
Max. Tire Force Rear (N)	2805	2785	2937	3105	3086
Nonlinear % of Front Spring Force	17	13	21	24	12
Nonlinear % of Rear Spring Force	15	30	29	29	17

5.5 Case 5

In this case, the fitness function was the sum of all the three terms (f_1, f_2 *and* f_3) where the main term was f_1 and the other two were set one order of magnitude lower. The main idea in this case was the use of the two targets more as penalties rather than targets of the optimization; this is the reason why f_2 *and* f_3 were one order of magnitude lower. First of all, regarding the effectiveness of the algorithms, the gradient based seems to concentrate to optimize only the term f_1, which depicts the ride comfort, ignoring the multi-objective character of this fitness function and failing to find a "good" optimal solution. This problem is also pointed out in the design variables of this optimization scenario (*S5*) in which some of them are trapped in the upper or lower bounds, as well as the time the problem needed to converge. The results of this case, regarding the gradient based algorithm, prove the conclusions indicated above regarding the effectiveness of the algorithm. As far as the other optimization scenarios are concerned, due to the multi-objective character of this case it is difficult to understand which optimization scenarios converged to the "best" optimal solutions. Moreover, in contrary with the previous case, neither the optimal design variables in every optimization scenario nor the table of the important parameters of vehicle's model could indicate similar characteristics in the optimal solutions.

Table 13 Optimal Solutions of Design Variables / Case 5

Design Variables	Optimization Scenarios				
	S1	S2	S3	S4	S5
K_{l_F}(N/m)	59559	49457	43581	37635	44761
C_F(N·s/m)	2414	2164	5312	2096	2000
K_{l_R}(N/m)	58680	56541	45694	65343	45798
C_R(N·s/m)	3901	6281	1910	4616	2000
K_{nl_F}(N/m^3)	$1.25*10^7$	$5.72*10^6$	$2.42*10^7$	$5.09*10^6$	$7.56*10^6$
K_{nl_R}(N/m^3)	$4.34*10^7$	$4.40*10^7$	$5.89*10^6$	$3.32*10^7$	$3.56*10^6$

Table 14 Vehicle Model's Parameters for the Optimal Solutions / Case 5

Vehicle Model's Parameters	Optimization Scenarios				
	S1	S2	S3	S4	S5
RMS(acc_{body}) (m/s^2)	0.605	0.587	0.538	0.538	0.466
Max. Suspension Travel Front (m)	0.038	0.040	0.026	0.038	0.040
Max. Suspension Travel Rear (m)	0.024	0.023	0.036	0.029	0.038
Max. Tire Force Front (N)	3556	3070	2829	2433	2836
Max. Tire Force Rear (N)	2549	2944	3154	3367	3302
Nonlinear % of Front Spring Force	23	16	27	16	21
Nonlinear % of Rear Spring Force	30	30	14	30	10

6. DISCUSSION

In order to compare all the optimal solutions retrieved with all the optimization Scenarios and all Cases, the optimal design points retrieved with Cases 1 - 3 and 5 were scaled to Case 4. Case 4 was selected as the ground Case due to its multi - objective character. In more details, the vehicle model was simulated with each set of the optimal design variables and the terms f_1, f_2 and f_3 have been recalculated and hence the optimal value of the fitnesses based on Case 4, leading to the comparison presented in Figure 6 and Table 15. In Figure 6, the fitness' value of each optimization scenario (*S1-S5 (different group of columns)*) is compared for all the cases (*Case 1-5 (columns of different color)*). In the Table of data of the figure, the optimum solution for every Case is pointed out in borders of the corresponding color. The comparison of the fitness' values is performed horizontally. In addition, Table 15 presents the values of the terms f_1,f_2 and f_3 as well as the sum of them for the optimum solution for each case, in addition with the optimization scenario in which it was found. The sum of the terms is the fitness values of the optimal solutions scaled to Case 4 and also presented in the data table of Figure 6.

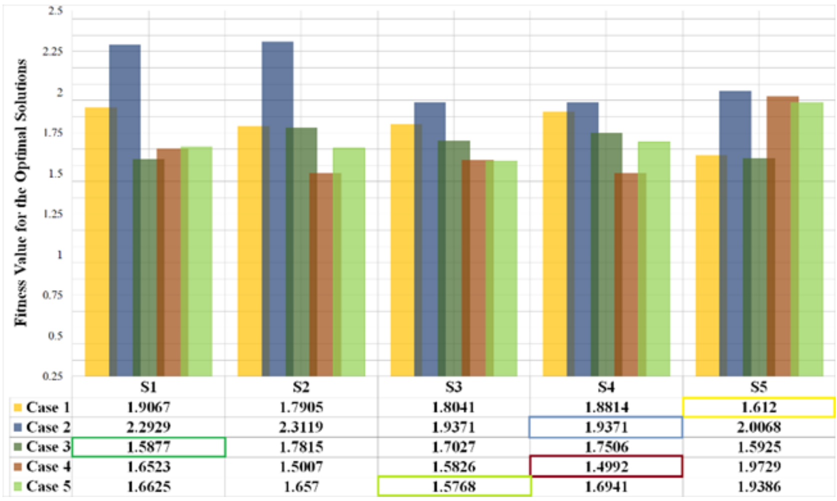


Figure 6 Comparsion of the Optimal Solutions based on the values of Optimal Fitnesses

Table 15 Comparison of the Fitness Values of the Optimal Solutions

Terms	Case 1	Case 2	Case 3	Case 4	Case 5
	S5	S4	S1	S4	S3
f_1	0.221	0.7921	0.4042	0.3466	0.2758
f_2	1.15	0.536	0.8685	0.8532	0.963
f_3	0.241	0.609	0.315	0.2994	0.338
$f_1+f_2+f_3$	1.612	1.9371	1.5877	1.4992	1.5768

Table 16 Comparison of the Design Variables of the Optimal Solutions

Design Variables	Case 1	Case 2	Case 3	Case 4	Case 5
	S5	S4	S1	S4	S3
K_{l_F}(N/m)	36194	38952	41279	42377	43581
C_F(N·s/m)	2582	8003	3632	4002	5312
K_{l_R}(N/m)	39741	47540	77718	60069	45694
C_R(N·s/m)	3228	8286	4319	3693	1910

K_{nl_F}(N/m3)	1.03*107	2.56*107	1.44*107	1.36*107	2.42*107
K_{nl_R}(N/m3)	1.18*107	1.60*107	5.00*107	2.86*107	5.89*106

At first, regarding the optimum solution of all the results, based on Table 15 and Figure 6, could be found in Case 4/S4 which was expected because Case 4 was the ground case for comparison of the optimal solutions. The next two near the optimum solution, were the one of Case 5/S3 as well as the one of Case 3/S1. Based on Table 15, the close values of the terms f_1, f_2 and f_3 of Case 4/S4 and Case 3/S1 show that they have converged to almost the same solution in combination with the optimal design variables of these solutions, as shown in Table 16. The only difference in the design variables of these two solutions is the stiffness of the rear suspension system which lead to the slightly higher term f_1, which depicts the ride comfort, in Case 3/S1 (*0.4042*) than Case 4/S4 (*0.3466*). On the other hand, Case 5/S3 appears to have converged to a different family of solutions. More specifically, in Case 5/S3 the rear suspension system is configured to lower values of the stiffness of the spring and of the damping coefficient than the optimal solutions of the other cases, as shown in Table 15, improving the ride comfort by decreasing term f_1 and increasing term f_2, representing the ride comfort and the suspension travel respectively, as shown in Table 16. Furthermore, Case 5/S3 achieved a solution better than this of the single objective of term f_1 (Case 1/S5), but slightly worse than the multi-objective with f_1, f_2 and f_3 balanced (Case 4/S4). This suggests that using the terms f_2 and f_3 as penalties, the optimization prioritizes the main term taking slightly into consideration the other two, leading to a middle ground solution. This is why the optimum solution of Case 5 is a combination of the characteristics of Case 1/S5 and Case 4/S4.

Regarding the optimization scenarios, the Hybrid Algorithm was the leading method either with population 200 (*S3*) or 1000 (*S4*), outnumbering the other methods, as shown in Table 15, where the three of the five optimal solutions are found through hybrid algorithms. Secondly, the gradient based algorithm proved its reliability in Case 1 and Case 3 in contrary to the other cases in which it failed. Based on the discussion in Cases 1-3, the gradient based algorithm (*S5*) could deliver acceptable results only when the RMS of vehicle’s body acceleration is connected directly and straight forward with the target of the optimization. This is the reason of its effectiveness in Case 3, where the fitness function, which depicts the average of both the front and rear tire deflections (f_3), increases or decreases according to the increase or the decrease of the RMS of vehicle’s body acceleration, which depicts the ride comfort (f_1), as shown in Figures 3b-5b.

As far as the different Cases are concerned, based on the data table of Figure 6, Case 3 seems to be superior than Case 1 due to the fact that the optimal values of the fitnesses in the line of Case 3 are always lower than the ones of Case 1, regardless the optimization scenarios. Moreover, in all the optimization scenarios, Case 3 not only delivered more satisfactory optimal solutions than Case 1 but also converged in much less computational time, lowering it by 10-40% depending on the optimization scenario. The argument regarding the superiority of Case 3, is validated from the fact that it delivered one of the most satisfactory optimal solution in comparison with the results of Case 4, as explained in the previous paragraphs in detail. Combining the above points, Case 3 converge to solutions taking into consideration not only the increase of the ride comfort but also the minimization of the suspension travel, indicating a multi-objective character despite having a single-objective fitness function. Moreover, based on Figure 6, Case 3 seems to be stable in providing satisfactory optimal results in comparison with all the other cases which sometimes fail to deliver depending the optimization scenario.

6. CONCLUSIONS

To sum up, in the current paper the optimization of a heavy vehicle's suspension system were investigated setting different optimization targets. Conclusions have been made not only regarding the fitness functions but also for the optimization methods used in order to reach the optimum solution more accurately and with less computational time. The remarks regarding Case 3, discussed in the previous section, outline the importance of tire deflection being a part of the fitness function due to the superiority of Case 3 over Case 1. Due to its multi-objective character, Case 3 seems to outnumber Case 1, which is the most common main term of fitness functions in literature as far as the optimization of suspension systems is concerned. This could lead to the use of the tire deflection, as the main target in the optimization of suspension systems in combination with various existing methods of the literature mentioned in the introduction. Furthermore, the effectiveness of the hybrid algorithm proved promising in comparison with the other algorithms. They were able to find more satisfactory solutions in most cases and with less computational time than the other algorithms. The most important regarding the hybrid algorithms is the need of finding the balance between the use of its genetic and gradient based parts so as to gain more from the hybridization and help the problem to converge in less time. Further work is in progress to extend our research.

REFERENCES

[1] Georgiou, G., Verros, G., & Natsiavas, S. (2007), "Multi-objective optimization of quarter-car models with a passive or semi-active suspension system", Vehicle System Dynamics, 45(1), pp 77-92.

[2] Özcan, D., Sönmez, Ü., & Güvenç, L. (2013), "Optimization of the nonlinear suspension characteristics of a light commercial vehicle", International Journal of Vehicular Technology, 2013.

[3] Shirahatti, A., Prasad, P. S. S., Panzade, P., & Kulkarni, M. M. (2008), "Optimal design of passenger car suspension for ride and road holding", Journal of the Brazilian Society of Mechanical Sciences and Engineering, 30(1), 66-76.

[4] Gündoğdu, Ö. (2007), "Optimal seat and suspension design for a quarter car with driver model using genetic algorithms", International Journal of Industrial Ergonomics, 37(4), 327-332.

[5] Alkhatib, R., Jazar, G. N., & Golnaraghi, M. F. (2004), "Optimal design of passive linear suspension using genetic algorithm", Journal of Sound and vibration, 275(3), pp 665-691.

[6] D. Koulocheris, H. Vrazopoulos and V. Detrimanis (2002), "Optimization Algorithms for tunning suspension systems used in ground vehicles", Proceeding of International Body Engineering Conference & Exhibition and Automotive & Transportation Technology Conferenece, Paris, France, July 2002, SAE Paper no. 2002-01-2214.

[7] Koulocheris D., Papaioannou G., Christodoulou D. (2016), "Multi objective optimization of a heavy vehicle nonlinear suspension system", Proceedings of the 11th International Congress of Mechanics (11^{th} HSTAM), Athens, Greece, 27-30 May 2016.

[8] Koulocheris D., Papaioannou G. (2016), "Experimental evaluation of the vertical wheel loads of a heavy vehicle validated with an optimized half car model", Proceedings of the 11th International Congress of Mechanics (11^{th} HSTAM), Athens, Greece, 27-30 May 2016.

[9] Koulocheris D., Papaioannou G. (2015), “Dynamic Analysis of the Suspension System of a Heavy Vehicle through Experimental and Simulation Procedure”, Proceedings of 25th International Automotive Conference “Science and Motor Vehicles” (25th JUMV), Beograd, Serbia, 14-15 April 2015.

[10] Sun, L., Cai, X., & Yang, J. (2007), “Genetic algorithm-based optimum vehicle suspension design using minimum dynamic pavement load as a design criterion”, Journal of Sound and Vibration, 301(1), 18-27.

[11] Uys, P. E., Els, P. S., & Thoresson, M. (2007), “Suspension settings for optimal ride comfort of off-road vehicles travelling on roads with different roughness and speeds”, Journal of Terramechanics, 44(2), pp 163-175.