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MULTI-CRITERIA OPTIMIZATION OF SINGLE INTERSECTION UNDER OVER-SATURATED CONDITIONS

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RESEARCH ARTICLE

ABSTRACT: Intersections are junctions where at least two roads or streets meet or cross. Consequently, they represent the most critical spots in the city traffic network, in terms of traffic safety. When determining the Level of Service at signalized intersections, the Highway Capacity Manual (HCM) does not take into account the traffic safety factors. In order to overcome this shortcoming, we propose in this paper the multi-criteria approach to the single intersection traffic control problem. The first objective function to be minimized is the average control delay experienced by all vehicles that arrive at the intersection. The second objective function to be minimized is the traffic safety risk index on intersection. These two criteria are in conflict, and the task is to find a good compromise between them. The analysed problem is solved by the combination of dynamic programming, and compromise programming. The proposed approach was tested in the case of the intersection that is controlled with two phases, under over-saturated traffic flows.

KEY WORDS: single signalized intersections, vehicles control delay, traffic safety risk index, dynamic programming, compromise programming

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VIŠEKRITERIJUMSKA OPTIMIZACIJA IZOLOVANE PREZASIĆENE RASKRSNICE

REZIME: Raskrsnice su mesta na uličnoj mreži gde su ukrštaju dva ili više saobraćajnih tokova. Kao posledica toga, one predstavljaju kritična mesta po pitanju bezbednosti saobraćaja. Kada se utvrđuje Nivo Usluge na signalisanim raskrsnicama, po HCMu (međunarodnom priručniku za ovu oblast), u obzir se ne uzimaju faktori bezbednosti saobraćaja. U ovom radu autori predlažu višekriterijumski pristup za rešavanje problema upravljanja izolovanom signalisanom raskrsnicom. Prvu kriterijumsku funkciju, koju treba minimizovati, predstavljaju ukupni prosečni vremenski gubici svih vozila koji prođu raskrsnicom u određenom periodu vremena. Druga kriterijumska funkcija, koju treba minimizovati, se odnosi na indeks rizika bezbednosti na raskrsnici. Kada su dve kriterijumske funkcije u konfliktu cilj je pronaći dobar kompromis između njih. Postavljeni problem je rešavan kombinacijom dinamičkog programiranja i kompromisnog programiranja. Predloženi pristup je testiran na primeru prezasićene raskrsnice kojom se upravlja pomoću dve faze.

KLJUČNE REČI: izolovana signalisana raskrsnica, vremenski gubici vozila, indeks bezbednosti, dinamičko programiranje, kompromisno programiranje

MULTI-CRITERIA OPTIMIZATION OF SINGLE INTERSECTION UNDER OVER-SATURATED CONDITIONS

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1. INTRODUCTION

An intersection represents the junction where at least two roads or streets meet or cross. There are different types of conflict points at traffic intersections. According to the number of traffic crashes occurring at intersections, it can be concluded that intersections are the most critical element at the street network. When determining signal plans (cycle and splits) engineers most frequently do not include the safety factor into account. Every traffic crash at the intersection results in a significant drop of capacity (up to 50% [9]). This percentage varies depending on the severity of the crash and the intersection clearance time after the crash. Additional consequences of reducing intersection capacity may be an increase in the congestion level in the network, an increase in the emission of harmful gases, an additional delay of vehicles, etc. For that reason, to efficiently control the signalized intersection, it is necessary to consider traffic safety factors.

In this paper, a mathematical model that optimizes two objective functions: the average control delay per vehicle and traffic safety risk index, is developed. The proposed mathematical model aims to provide such signal plans (cycle and splits) that control traffic demand in an efficient and safe manner.

Controlling the single intersection was an area of interest for many authors in the last few decades. The most common objective functions used in various models are the total vehicle delays, the total number of stops, or some of the economic and environmental criteria. The author [2] proposes one of the first models for controlling a single signalized intersection. Webster's model determines the cycle time and the split of green time for under-saturated traffic demands without phases overlap. Some of the most important works are as follows: ([1, 2, 3, 6, 8, 12, 13, 15, 17]).

The problems of controlling an over-saturated single intersection were analyzed, among others, by the following authors ([4, 18]).

Analysis of the traffic control problems, taking into account traffic safety factors, can be found in the following papers ([9, 16, 19]). Authors [16] developed a unique fuzzy logic model for controlling traffic flows in specific situations at the intersections, such as traffic crash, construction zone at the intersection and so on. Authors [9] proposed a new methodology for determining the Level of Service at a signalized intersection taking into account the traffic safety risk index. The methodology assumes that the safety risk index is in the function of the control parameters (cycle and splits) at the intersection. Authors [19] developed a multi-criteria model based on genetic algorithms. The aim of the author was to optimize two objective functions at the same time: vehicle control delay and safety of vehicles and pedestrians at the intersections. Other researches that treat the traffic control problems from the aspect of traffic safety can be found in the following papers ([5, 14, 22, 24]).

2. FORMULATION OF THE PROBLEM

The problem that we study in this paper could be formulated in the following way: For the given number of phases, determine cycle length, and splits in such a way to minimize the two objective functions at the same time: average control delay experienced by all vehicles that arrive at the intersection within given time period, and the traffic safety index under over-saturated conditions.

Let us first analyze the formulas to calculate the average vehicle control delay in oversaturated conditions, as well as the traffic safety indexes at a single signalized intersection. In the next step, we propose the mathematical formulation of the problem.

2.1 Vehicle control delay under over-saturated conditions

Let us suppose that we have parameters of the signal plan: cycle (C) and the green time of the i-th line or i-th group of lines (g_i). The average control delay per vehicle on the i-th line or the i-th group of lines at the signalized intersection (d_i) are calculated as (HCM 2010):

$$d_i = \frac{0.5 \cdot C \cdot \left(1 - \frac{g_i}{C}\right)^2}{1 - \left[min(1, X_i) \cdot \frac{g_i}{C}\right]} + 900T\left[(X_i - 1) + \sqrt{(X_i - 1)^2 + \frac{4 \cdot X_i}{c_i T}}\right] + d_{3i} \tag{1}$$

where:

X_i - degree of saturation on i-th line or i-th group of lines,

c_i - capacity of i-th line or i-th group of lines (veh/h),

T - duration of analysis period (h),

d_{3i} - additional delay, per vehicle, on i-th line or i-th group of lines (s/veh).

The relation between traffic demand (q_i) and capacity (c_i), on i-th line or i-th group of lines, present degree of saturation (X_i), i.e. (HCM 2010):

$$X_i = \frac{q_i}{c_i} = \frac{q_i}{s_i \frac{g_i}{C}} = \frac{q_i / s_i}{g_i / C} \tag{2}$$

where s_i represent saturation flow for i-th line or i-th group of lines.

The additional delay d_{3i} equals (HCM 2010):

$$d_{3i} = \frac{1800 \cdot Q_{bi} \cdot \ (1 + u_i) \cdot t_i}{c_i \cdot T} \tag{3}$$

where Q_{bi} is initial queue (veh) on the i-th lane or i-th group of lines; c_i is capacity of the i-th lane (veh/h) or i-th group of lines; t_i is duration of unmet demand of the i-th lane or i-th group of lines; u_i is delay parameter of the i-th lane or i-th group of lines.

The duration of unmet demand, t_i, and delay parameter, u_i, equals:

$$t_i = min\left\{T, \frac{Q_{bi}}{c_i \cdot [1 - min(1, X_i)]}\right\} \tag{4}$$

$$if\ t_i < T\ then\ u_i = 0; else\ u_i = 1 - \frac{c_i \cdot T \cdot [1 - min(1, X_i)]}{Q_{bi}} \tag{5}$$

2.2 Traffic safety risk index

Traffic safety risk index on intersection during the j-th phase (RI_j) is calculates as [9]:

$$RI_j = \frac{g_j + y_j}{C} \cdot \sum_k NCV_k \cdot SCP_k \quad (6)$$

where:

k - index of conflict type at intersection,

g_j - green time of j-th phase,

y_j - yellow time of j-th phase (3 second),

NCV_k - number of vehicles in conflict for k-th type,

SCP_k - degree of "severity" for k-th type.

At a single intersection, exist three types of vehicle-to-vehicle traffic conflicts: 1. Crossing (k=1); 2. Merging (k=2); 3. Diverging (k=3). We assume that for different types of conflict, degree of "severity" have the following values: SCP_1=3; SCP_2=1.5; SCP_3=1.

2.2.1 Mathematical formulation of the problem

The mathematical formulation of the traffic signal timing problem in the case of over-saturated flows reads:

$$\mathit{Minimize} \qquad d = \frac{\sum_i q_i \cdot d_i}{\sum_i q_i} \quad (7)$$

$$\mathit{Minimize} \qquad RI = \sum_j RI_j \quad (8)$$

$$\mathit{Subject\ to}: \qquad C_{min} \le C \le C_{max} \quad (9)$$

$$g_{jmin} \le g_j \le g_{jmax}, \qquad \forall\, j \in F \quad (10)$$

$$\sum_j g_j = C - L \quad (11)$$

where:

d – average control delay at the intersection (s/veh),

RI - traffic safety risk index at the intersection,

w_1, w_2 – objective function weights,

F - total number of phases,

L - Cycle lost time (seconds),

C_{min}, C_{max} - minimal and maximum values for cycle (seconds),

g_{jmin}, g_{jmax} - minimal and maximum green times for j-th phase (seconds).

Criteria function (7), to be minimized, presents the average control delay experienced by all vehicles that arrive at the intersection within a given time period under over-saturated conditions. Criteria function (8), to be minimized, presents and traffic safety risk index at the intersection. The constraint (9) defines the interval of the feasible cycle length values. The constraint (10) defines the interval of the feasible green time length values. The relationship between cycle length, green time lengths and the lost time is described by the constraint (11).

3. THE PROPOSED SOLUTION OF THE PROBLEM BY COMPROMISE PROGRAMMING AND DYNAMIC PROGRAMMING

The chosen objective functions are mutually in conflict. There is usually no solution that at the same time minimizes both criteria. For this reason, the solution of the considered problem usually comprises a Pareto-optimal solution. This is the solution where no criterion can be improved without simultaneously worsening at least one of the other criteria.

The x^* is the Pareto optimal solution for problem if there is no other $x \in X$ such that:

$$f_i(x) \leq f_i(x^*), \qquad \forall\, i = 1, 2, \dots, r \tag{12}$$

The index i is the index of the objective functions and goes to r (in our case, $r = 2$).

In order to solve the multi-criteria optimization problem, we use the Compromise Programming. By using the Compromise Programming we try to obtain a solution that is as close as possible to the ideal solution in terms of distance. We try to minimize the L_p distance (Duckstein 1984) that is defined in the following way:

$$L_p = \left[\sum_{i=1}^{r} {w_i}^p \cdot \left|\frac{f_i(\vec{x}) - f_i^o}{f_{imax} - f_i^o}\right|\right]^{\frac{1}{p}} \tag{13}$$

where:

-$f_i(\vec{x})$ -i-th objective function value that is the result of implementing decision $\vec{x}$,

f_i^o- the optimum value of the i-th objective function,

f_{imax} - the worst value obtainable for the i-th objective function,

w_i - i-th objective function's weight,

p - the value that shows distance type.

For $p = 1$, all deviations from the optimal solutions are in direct proportion to their magnitude, while when $2 \leq p \leq \infty$, a bigger deviation carries a larger weight in the L_p metric.

In the our case, we calculate the L_p in the following way:

$$L_p = \sqrt{w_1^2 \cdot \left|\frac{d - d_{opt}}{d_{worst} - d_{opt}}\right| + w_2^2 \cdot \left|\frac{RI - RI_{opt}}{RI_{worst} - RI_{opt}}\right|} \tag{14}$$

where w_1 is the weight of the first criteria and w_2 is the weight of the second criteria. The optimal values for the first and the second criteria are denoted by d_{opt} and RI_{opt}, respectively.

In order to obtain the optimal values, as well as the worst values we perform the single-objective optimization. We obtain the solution that produces the optimal delay value by solving the problem (7) - (11) and by ignoring the objective function (8). This optimal solution generates simultaneously the worst traffic safety risk index. The optimal traffic safety risk index and the worst delay could be obtained by solving the problem (7) – (11) and by ignoring the objective function (7).

We perform the single-objective optimization, as well as the minimization of the L_p metric, by using the Dynamic Programming technique. The Dynamic Programming splits the problem into several, simpler, sub-problems. Dynamic Programming is a mathematical procedure that solves problem in stages. The computations at the various stages are linked in such a way, that the optimal solution of the considered problem is obtained when the final stage is reached. The basic concept of the Dynamic Programming is most easily explained by the network created of stages and states. A network suited for traffic control on a signalized isolated intersection is shown in the Figure 1. Let us introduce the following notation:

- g_1 - green time allocated to phase 1,
- g_2 - green time allocated to phases 1 and 2,
- g_n - green time allocated to phases 1, 2,.. and *n*-th.

Any of the stages can be found in some of the states. Stages in this network model, except the first one, represent the signal phases. The first stage represents the cycle value reduced by all red time (L). States for the first stage take a value from range C_{min} to C_{max} with a 1-second increment. Other stages (g_1, g_2, to g_n) can be found in some of the states, from g_{min} to g_{max}, with a 1-second increment.

The network is created only by those branches that connect compatible nodes. Compatible nodes are those which satisfy the constraints (11). The branches are weighted by appropriate metric. Finding the shortest path through the network gives us an optimal solution for the analyzed problem.

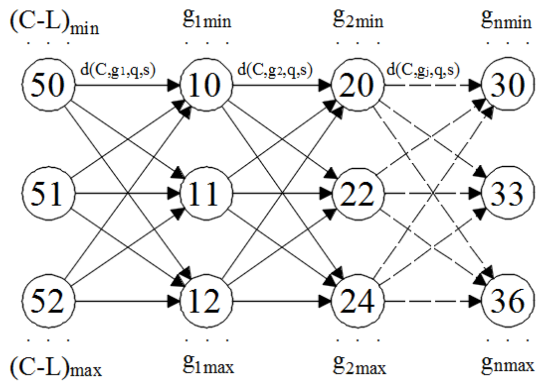

Figure 1. Dynamic programming network

The inefficiency of this algorithm is reflected in the long CPU time. Practically, it can only be applied up to signal plan with three phases [10].

4. NUMERICAL EXAMPLE

The proposed approach for solving the analyzed problem was tested on the hypothetical intersection shown in the Figure 2. We analyzed the over-saturated intersection controlled by two phases. The lane flows, for the considered intersection are indicated in Figure 2 (in veh/h). The analyzed period is equal to one hour (T = 1 hr). Cycle lost time L is equal to 12 (in seconds). Table 1 shows saturation flow values per line that were used in the calculation. The value of the initial queues of unserved vehicles, at the beginning of time period T, is also given in Table 1. Traffic lines are indicated with capital letters A, B, C and so on.

Table 1. Lane saturation flows and initial queues

Saturation flows (veh/h)											
A	B	C	D	E	F	G	H	L	M	R	T
1500	1600	870	451	1600	1500	456	1600	1500	1500	1600	613
Initial queues (veh)											
A	B	C	D	E	F	G	H	L	M	R	T
21	10	17	0	12	5	0	4	7	14	8	2

The minimum allowed cycle length C_{min} and the maximum allowed cycle length C_{max} were respectively equal to 60 seconds and 120 seconds, while the minimum value of the green time g_{min} and the maximum value of the green time g_{max} were respectively equal to 7 seconds and 80 seconds.

These input data are taken from the paper of [11].

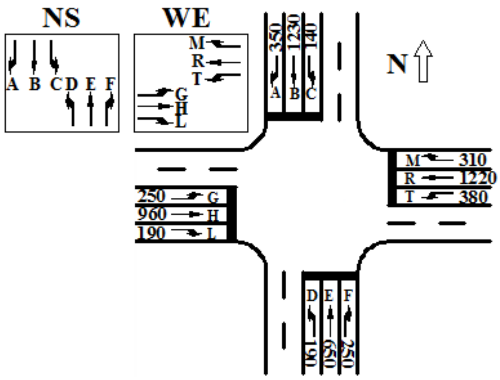

Figure 2. Test intersection with phase plan and traffic demands

The obtained results are shown in the Table 2. We presented the obtained solutions as the following sequence: C, g_1, g_2, ..., g_n. For example, the solution 120, 50, 58 represents the case when the cycle length equals 120 seconds. There are two phases in this case. The green light lengths are respectively equal 50 seconds and 58 seconds.

Table 2. Solutions

w		Solution	d (s/veh)	RI	L_p
w_1	w_2				
1	0	120;50,58	697.42	8691.25	0
0	1	60;41,7	4137.95	7572.50	0
0,9	0,1	80;34,34	829.08	8394.38	0.087
0,8	0,2	60;25,23	983.26	8132.50	0.127
0,7	0,3	60;28,20	1120.42	8027.50	0.155
0,6	0,4	60;30,18	1264.94	7957.50	0.175
0,5	0,5	60;32,16	1462.23	7887.50	0.184

0,4	0,6	60;34,14	1731.93	7817.50	0.182
0,3	0,7	60;36,12	2116.10	7747.50	0.168
0,2	0,8	60;38,10	2669.10	7677.50	0.139
0,1	0,9	60;40,8	3519.24	7607.50	0.088

Like in other practical multicriteria decision-making problems, a solution must be found that is frequently called the "implementation" solution. In order for a solution to be accepted as the best from the users' viewpoint, the decision maker must have other solutions for comparison.

In our case, because of the over-saturated conditions, the "implementation" solution could be the solution where *w* is (0.9, 0.1).

5. CONCLUSION

The mathematical model presented in this paper tries to optimize the control parameters of the single signalized intersection: cycles and splits, under over-saturated conditions. A multi-criteria approach has been proposed based on the delays of vehicles and traffic safety indexes at the intersection. Dynamic programming is applied as a method for finding optimal solutions in the case of single objective function. Compromise programming was used as a method for solving the multi-criteria optimization problem. The model was tested on a hypothetical intersection that is controlled by two phases. In the case where an intersection is controlled with more than three phases, this algorithm becomes ineffective. In cases with more than three phases some of the metaheuristics algorithms should be used instead Dynamic Programming. The numerical example shows the applicability of the proposed approach.

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REFERENCES

[1] Akcelik, R.: “Traffic signals: capacity and timing analysis”, Research Report 123, Australian Road Research Board, Melbourne, Australia, 1981.

[2] Bell, M.G.H.: “A probabilistic approach to the optimization of traffic signal settings in discrete time”, 11th International Symposium on Transportation and Traffic Theory, Tokyo, 1990, pp. 619-631.

[3] Brilon, W., Wu, N.: “Delays at fixed-time traffic signals under time-dependent traffic conditions”, Traffic Engineering and Control, Vol. 31, No. 12, 1990, pp.623–631.

[4] Chang, T. H., Lin, J. T.: „Optimal signal timing for an oversaturated intersection“, Transportation Research Part B, Vol. 34, No. 6, 2000, pp.471-491.

[5] Cunto, F., Saccomanno, F.F.: “Calibration and validation of simulated vehicle safety performance at signalized intersections”, Accident analysis & prevention, Vol. 40, No. 3, 2008, pp.1171-1179.

[6] Dresner, K., Stone, P.: 2006. “Traffic intersections of the future”, In Proceedings of the National Conference on Artificial Intelligence, Menlo Park, CA; Cambridge, MA; London; AAAI Press; MIT Press; Vol. 21, No. 2, 1999, p. 1593.

[7] Duckstein, L.: „Multiobjective optimization in structural design:the model choice problem“, In: E. Atrek, R.H. Gallagher, K.M. Ragsdell, and O.C. Zienkiewicz, eds. New directions in optimum structural design. New York: John Wiley, 1984, pp.459-448.

[8] Duisters, K.S.: “Formulating and testing an algorithm for fixed time control of traffic intersections”, Internship report, Eindhoven University of Technology, 2013.

[9] Fuquan, P., Jian, L., Qiaojun, X., McAvoy, D.S.: “Safety Based Signalized Intersection Level of Service”, In Transportation and Development Innovative Best Practices, 2008, pp. 246-251.

[10] Jovanović, A.: “Choice of signal timing for traffic control by bee colony optimization”, Doctoral dissertation, University of Belgrade, Faculty for transport and traffic engineering, 2017.

[11] Jovanović, A.**,** Teodorović, D.: „Pre-timed control for under-saturated and over-saturated isolated intersection: A Bee Colony Optimization (BCO) approach“, Transportation Planning and Technology, 2017.

[12] Kronborg, P., Davidsson, F.: “MOVA and LOHVRA: traffic signal control for isolated intersections”, Transportation Research Board, Vol. 34, No. 4, 1993, pp.195-200.

[13] Lan, C.: “New optimal cycle length formulation for pretimed signals at isolated intersections”, Journal of Transportation Engineering, Vol. 130, No. 5, 2004, pp.637–647.

[14] Leden, L.: “Pedestrian risk decrease with pedestrian flow”, A case study based on data from signalized intersections in Hamilton, Ontario. Accident Analysis & Prevention, Vol. 34, No. 4, 2002, pp.457-464.

[15] Murat, S., Gedizlioglu, E.: “A fuzzy logic multi-phased signal control model for isolated junctions”, Transportation Research Part C, Vol. 13, No. 1, 2005, pp.19-36.

[16] Nair, B.M., Cai, J.: “A fuzzy logic controller for isolated signalized intersection with traffic abnormality considered”, In 2007 IEEE intelligent vehicles symposium, IEEE, 2007, pp. 1229-1233.

[17] Pappis, C.P., Mamdani, E.H.: „A fuzzy logic controller for a traffic junction”, Transactions on Systems, Man and Cybernetics Vol. 7, No. 10, 1977, pp.707–717.

[18] Park, B.: „Traffic signal optimization program for oversaturated conditions: genetic algorithm approach“, Transportation Research Record: Journal of the Transportation Research Board, Vol. 1683,1999, pp.133-142.

[19] Stevanović, A., Stevanović, J., Kergaye, C.: “Optimization of traffic signal timings based on surrogate measures of safety”, Transportation research part C: emerging technologies, Vol. 32, 2013, pp.159-178.

[20] TRB, H.C.M., HCM, Transportation Research Board–special report, 209, 2010.

[21] Webster, F.V.: “Traffic signal settings”, Road Research Technical Paper, Road Research Laboratory, Her Majesty Stationary Office, London, UK, No. 39, 1958.

[22] Xie, K., Wang, X., Huang, H., Chen, X.: “Corridor-level signalized intersection safety analysis in Shanghai”, China using Bayesian hierarchical models. Accident Analysis & Prevention, Vol. 50, 2013, pp.25-33.

[23] Zhao, L.: "A fast signal timing algorithm for individual oversaturated intersections", IEEE Transactions on Intelligent Transportation Systems, Vol. 12, No. 1, 2011, pp.280-283.

[24] Zhang, L., Prevedouros, P.: "Signalized intersection level of service incorporating safety risk", Transportation Research Record: Journal of the Transportation Research Board, Vol. 1852, 2003, pp.77-86.