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APPLICATION OF MULTI-PARAMETER FREQUENCY ANALYSIS IN EXPERIMENTAL IDENTIFICATION OF VIBRATION PARAMETERS IN MOTOR VEHICLES

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RESEARCH ARTICLE

ABSTRACT: During exploitation, motor vehicles are exposed to vibrational loads that cause fatigue for users and material fatigue of their components. Therefore, vibrations must be studied at the earliest stage of design, using mathematical models, experiments, or a combination of both. Generally, there are idealizations during theoretical analysis, particularly concerning operating conditions and the interrelationships of motor vehicle components. This paper attempts to develop a method for identifying actual loads in continuous vehicle systems in exploitation conditions, based on recorded spatial vibrations. Multi-parameter frequency analysis was used for 2D, 3D, and 4D Fourier transformations. An illustration of the application of these procedures was carried out on idealized continuous vehicle elements such as bars, membranes, and parallelepipeds.

KEY WORDS: *Vehicle, continuous systems, vibrations, multi-parameter frequency analysis*

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PRIMENA VIŠEPARAMETARSKE FREKVENTNE ANALIZE PRI EKSPERIMENTALNOJ IDENTIFIKACIJI PARAMETARA VIBRACIJA MOTORNIH VOZILA

REZIME: Motorna vozila su, tokom eksploatacije, izložena vibracijskim opterećenjima koja dovode do zamora korisnika i materijala njihovih agregata. Zbog toga se vibracije moraju izučavati još u najranijoj fazi projektovanja, uz korišćenje matematičkih modela, eksperimenata, ili njihovih kombinacija. U teorijskim razmatranjima se, obično, čine idealizacije, naročito u pogledu eksploatacionih uslova i međusobnih veza agregata motornih vozila. U ovom radu je učinjen pokušaj razvoja metode za identifikaciju stvarnih opterećenja kontinualnih sistema vozila u eksploatacionim uslovima, na osnovu registrovanih prostornih vibracija. Naime, za višeparametarsku frekventnu analizu korišćene su 2D, 3D i 4D Furijeove transformacije. Ilustracija mogućnosti primene tih postupaka je izvršena na idealizovanim kontinualnim elementima vozila (štapovi, membrane i paralelopipedi.

KLJUČNE REČI: *Vozilo, kontinualni sistemi, vibracije , višeparametarska frekventna analiza*

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INTRODUCTION

During exploitation, motor vehicles are exposed to vibrational loads that cause fatigue for users and material fatigue of their components. Therefore, vehicle vibrations must be studied at the earliest stage of their design using mathematical models, experiments, or a combination of both. Theoretical considerations usually focus on the vibration of concentrated masses, but with the development of numerical methods, especially finite element methods [1-3], attention has also been paid to the vibration of continuous vehicle systems.

Generally, there are idealizations, particularly concerning operating conditions [1-6] and the interrelationships of motor vehicle components. Then, usualy, idealizations are made, especially in terms of exploitation conditions and interconnection of motor vehicle aggregates [1]. The specific nature of motor vehicle exploitation conditions is their random nature, making theoretical considerations challenging to deal with using models, making experiments practically irreplaceable. Despite significant progress in the development of software for the automatic design and calculation of motor vehicles [7], their final characteristics are determined based on experimental studies.

Experimental methods are still relevant, particularly when considering continuous vehicle systems that are subject to vibration, such as different shafts, plates, support systems, chassis, and others. In this regard, methods for identifying their vibration parameters have been developed, such as modal analysis [8,9]. These methods, practically carried out in laboratory conditions, determine the vibration modes.

However, a problem arises when actual operating conditions are necessary for generating loads on shakers in the laboratory since modal analysis does not provide opportunities for generating these signals. Therefore, it is essential to develop a procedure for identifying vibration parameters that will enable their generation in laboratory conditions.

One possibility is frequency analysis using the Fourier transformation, which allows the determination of the frequency content of a signal by calculating magnitude and phase spectra. The data on magnitude spectra and phase angles [10], along with the inverse Fourier transformation, enables the generation of the original time-variant signal, which is routinely performed when the signal depends only on time.

However, the vibration of continuous systems depends on several parameters, such as size and time. Therefore, multi-parameter Fourier transformation must be used (2D, 3D, 4D...)[1-15] depending on the degree of simplification (idealization) of the problem. For the analysis of bar vibration, 2D transformation can be used, while for flat plates, 3D, and for spatial continuous bodies, 4D Fourier transformation can be used.

This paper analyses the possibilities of applying multi-parameter Fourier transformations to create the conditions for investigating the vibration of continuous vehicle systems in laboratory conditions.

$$F(\xi_1,\xi_2....\xi_n)=\int_{R^n} e-2\pi i(x_1\zeta_1+x_2\zeta_2+...+x_n\zeta_n)* \\ f(x_1,x_2....x_n)dx_1dx_2 ****dx_n \tag{1}$$

where:

- function of n variables,
- variables,
- circular frequency, and
- multiple integral (for 2D-double, 3D-triple, etc.).

1. METHOD

As previously mentioned, this paper aims to explore the possibility of applying multi-parameter frequency analysis (Fourier transformation: 2D, 3D, 4D) in the identification of vibration parameters of motor vehicles. Considering that the continuous elements and aggregates of motor vehicles can be simplified by modelling them as continuous bars (shafts, etc.), membranes (body parts...), and spatial bodies (engine, transmission, wheels...), it is deemed appropriate to explain the procedure on elementary elastic elements: bars, membranes, and parallelepipeds, using the multi-parameter Fourier transformation.

In the absence of experimental data on registered vibrations of mentioned continuous bodies (bar, membrane, parallelepiped), the method is illustrated with data obtained from mathematical models. As it is known, vibrations of continuous elements are described by partial differential equations [17]. Since performing partial differential equations that describe vibrations of observed continuous bodies are described in detail in [11,17], this will not be done here, but their final form will be given. For further consideration, images 1 to 3 will be observed.

Forced longitudinal vibrations of the bar

Forced longitudinal vibrations of the bar [11,17] are described by the differential equation:

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}+f(x,t) \tag{2}$$

where:

- u(x,t) - longitudinal vibrations of the bar,
- x - coordinate along the length of the bar,
- $f(x,t)$ - the force,
- t - time, and

$$c^2=\frac{E}{\rho}$$

where:

- E – Young’s modulus , and

- ρ – Membrane material density.

Forced transverse vibrations of the membrane

The differential equation that describes forced transverse vibrations of a rectangular membrane is of the form [11,17]:

$$\frac{\partial^2 u}{\partial t^2} = s^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + f(x, y, t), \tag{3}$$

where:

- u(x, y, t) - transverse vibrations of the membrane,
- x - coordinate along the membrane length,
- y - coordinate across the membrane width,
- f(x, y, t) - force,
- t - time, and

$$s^2 = \frac{\sigma}{\rho}$$

where:

- σ-axial stress, and
- ρ – membrane material density.

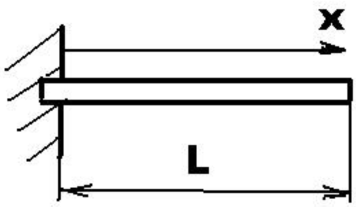


Figure 1. The continuous bar

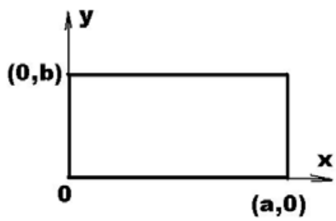


Figure 2. Rectangular continuous membrane

1.1 Forced spatial vibrations of continious parallelopipe

There are several forms of partial differential equations in the references that describe forced vibrations of a continuous parallelepiped, but it is considered useful to adopt the form [11,17] for further analysis:

$$\frac{\partial^2 u}{\partial t^2} = s^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + f(x, y, z, t), \tag{4}$$

where:

- u(x, y, z, t) - spatial vibrations of the Parallelepiped,
- x - coordinate along the length of the Parallelepiped,
- y - coordinate across the width of the parallelepiped,
- z - coordinate along the height of the Parallelepiped,
- f(x, y, z, t) - force,
- t - time, and
- $s^2 = \frac{\sigma}{\rho}$
- σ-axial stress, and
- ρ - mass density of the Parallelepiped.

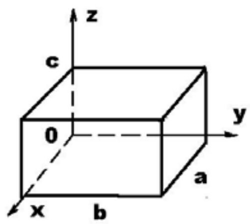


Figure 3: The continuous parallelepiped

As it is known [11,12,17,18], to find the general integral of partial differential equations (2-4), boundary and initial conditions must be known, as well as forces, which are shown in Table 1 in this specific case.

Table 1. The Boundary, initial conditions, and forced force

Bar	Membrane	Parallelepiped
$u(0,t)=0$ $\frac{\partial u(L,t)}{\partial x}=0$ $u(x,0)=0$ $\frac{\partial u(x,0)}{\partial t}=0$ $f(x,t)=a_m*\sin(4\pi t)$ $a_m=1,mm$	$u(0,y,t)=0$ $u(a,y,t)=0$ $u(x,0,t)=0$ $u(x,b,t)=0$ $u(x,y,0)=0$ $\frac{\partial u(x,y,0)}{\partial t}=0$ $f(x,y,t)=a_m*(random-0.5)$ $a_m=1,mm$	$u(0,y,t)=0$ $u(a,y,t)=0$ $u(x,0,z,t)=0$ $u(x,b,z,t)=0$ $u(x,y,0,t)=0$ $u(x,y,c,t)=0$ $u(x,y,z,0)=0$ $\frac{\partial u(x,y,z,0)}{\partial t}=0$ $f(x,y,z,t)=a_m*(random-0.5)$ $a_m=1,mm$

To verify the accuracy of the integration method for the bar, a harmonic force was used, while for the membrane and parallelepiped, very rigorous excitation forces with "white" noise were used.

The partial differential equations (2-4) are only possible to be solved in the case of the bar due to the harmonic force, while for the membrane and parallelepiped, it was not possible due to the random nature of the force. Therefore, an attempt was made to solve the partial differential equations using the program Wolfram Mathematica 13.2 [16]. However, difficulties arose in listing numerical data, particularly for the membrane and parallelepiped.

As a result, the problem was solved numerically [19] using the finite difference method with software developed in Pascal for the case of 2D, 3D, and 4D Fourier transform. Using the developed softwares, vibration values were calculated in all three cases with data (bar: l=256mm, h_x=1mm, ht=0.01s; membrane: $a=b$=128mm, h_x= h_y= 8mm, h_t=0.1s; parallelepiped: $a=b=c$=128mm, $h_x=h_y=h_z$=16mm, h_t=0.2s).

It is noted that in the case of analyzing the vibration of the bar, the solution depends on two parameters (which requires the use of 3D graphics) and in the case of the membrane and parallelepiped, it is necessary to use 4D or 5D graphics (which commercially does not exist). To use existing commercial 3D graphics, only the results for the center of gravity axes are shown for the membrane and parallelepiped. For illustration, the results of the numerical integration of the partial differential equation (2) are shown in Figures 4-6.

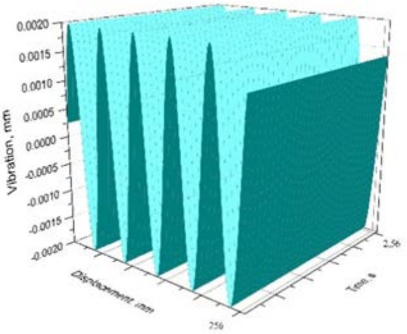


Figure 4. The longitudinal vibrations of the bar

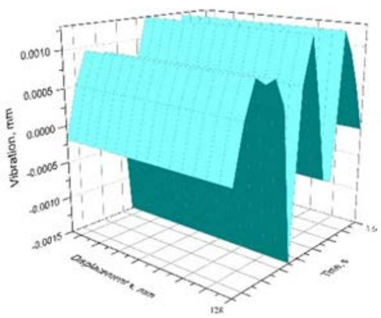


Figure 5. The longitudinal vibrations of the membrane in the "x" direction of the center of gravity axe

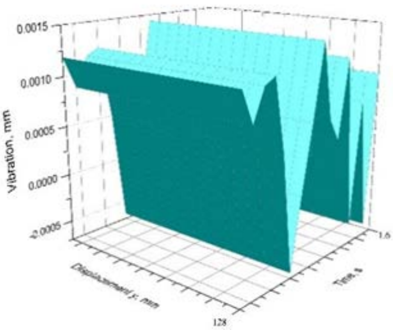


Figure 6. The longitudinal vibrations of the membrane in the “y” direction of the center of gravity axe

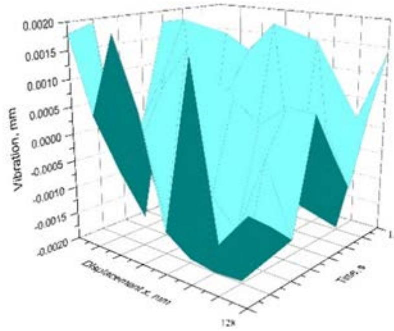


Figure 7. The vibrations of a parallelepiped in the” x “direction of the center of gravity axe

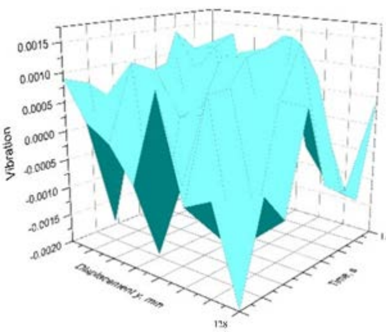


Figure 8. The vibrations of a parallelopiped in the “y” direction of the center of gravity axe

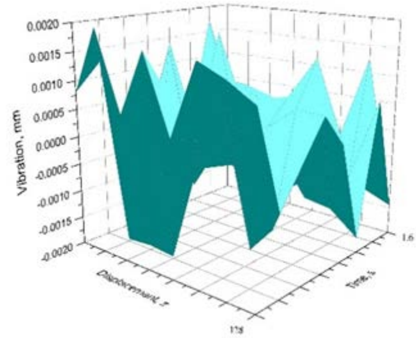
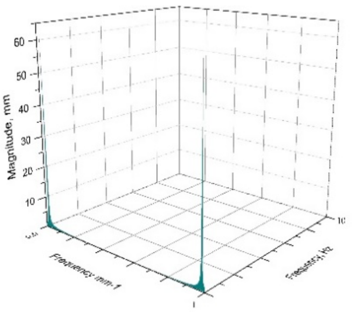


Figure 9. The vibrations of the parallelopiped in the ”z” direction of the center of gravity axe

Analysis of Figure 4 shows the harmonic character of the vibrations that move along the length of the bar in the form of waves because it is a test with harmonic excitation, which is similar with theoretical solutions from [11, 17]. Longitudinal vibrations of the bar depending on the displacement of “*x*” and time “*t*”. Therefore, the 2D Fourier transform must be applied, and for illustration, graphs of the spectrum magnitude and phase angle of vibrations are shown in Figures 10 and 11.



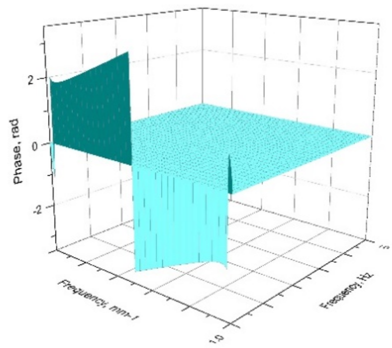
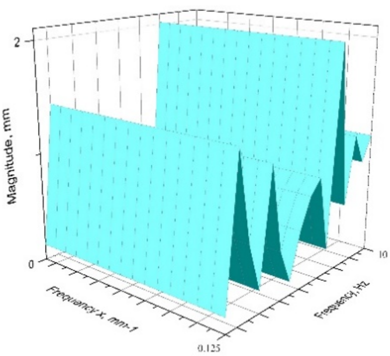


Figure 10. Spectrum magnitude of the longitudinal vibration of the bar

Figure 11. Phase angle of longitudinal vibrations of the bar

Transverse vibrations of the membrane spread wave-like over its surface and depend on three parameters: displacement along the "x" and "y" axes and time "t". In this case, it is necessary to apply the 3D Fourier transform, for which graphic display requires 4D graphics. To solve the problem, only data of vibrations along the center of gravity axes are shown in Figures 12-15.



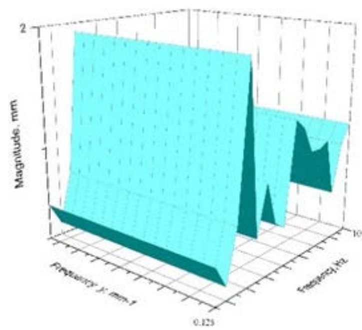


Figure 12. The spectrum magnitude of transverse vibrations of the membrane for the " x" center of gravity axe

Figure 13. The spectrum magnitude of transverse vibrations of the membrane for the" y"center of gravity axe

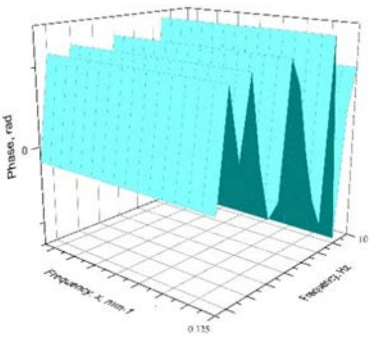


Figure 14. The phase angle of the spectrum of transverse vibrations of the membrane for the "x" center of gravity axe

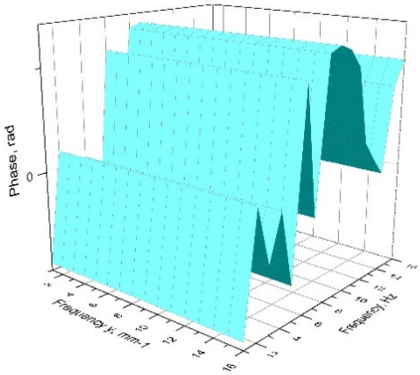


Figure 15. The phase angle of the spectrum of transverse vibrations of the membrane for the "y" center of gravity axe

Vibrations of parallelepiped spread throughout its volume and depend on 4 parameters: displacement in the "x", "y", and "z" directions and time" t". Frequency analysis requires the use of 4D Fourier transform and graphic display of 5D graphics. Therefore, it has been deemed appropriate to show only the results along the center of gravity axes, as shown in Figures 16-21.

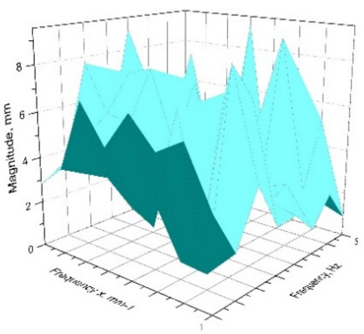


Figure 16. The spectrum magnitude of spatial vibrations of parallelepiped for the "x" center of gravity axe

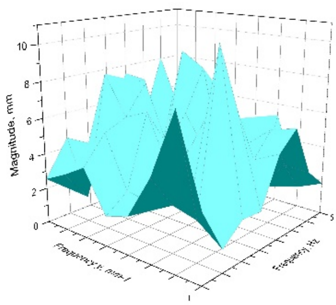


Figure 17. The spectrum magnitude of spatial vibrations of parallelepiped for the "y" center of gravity axe

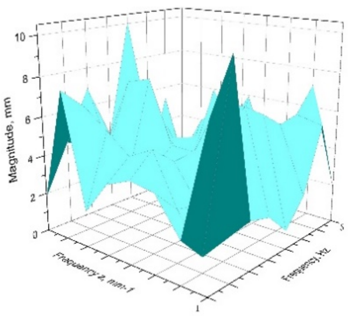


Figure 18. The spectrum magnitude of spatial vibrations of parallelepiped for the" z "center of gravity axe

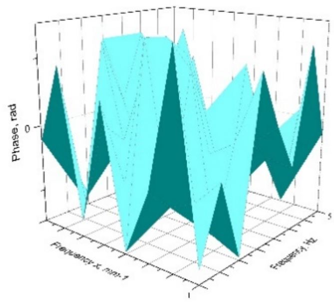


Figure 19. The phase angle of the spectrum of vibrations of parallelepiped for the "x" center of gravity axe

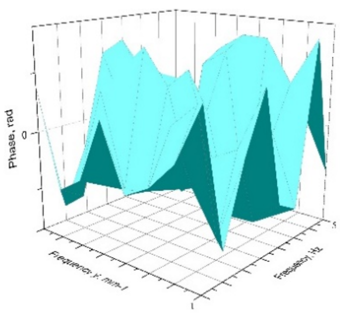


Figure 20. The phase angle of the spectrum of vibrations of parallelepiped for the "y" center of gravity axe

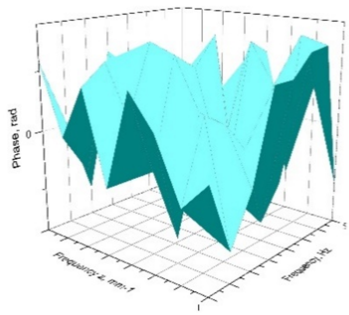


Figure 21. The phase angle of the spectrum of vibrations of parallelepiped for the "z" center of gravity ax

2. DATA ANALYSIS

By analysing the partially displayed data in Figures 10-21, it can be concluded that the spectrum magnitudes and phase angles describe the shape of vibrations in observed continuous bodies.

In the case of the bar, considering the harmonic nature of the excitation, the harmonics along its length (figures 9-11) are more noticeable, which is also theoretically confirmed in [11,17].

Data on transverse vibrations of the membrane, partially displayed in figures 12-15, show that harmonics occur across the surface of the membrane, regardless of whether the excitation was random, which agreed with theoretical interpretations related to transverse vibrations of the membrane [11,17].

By analysing all calculated data on spatial vibrations, partially shown in figures 16-21, extremes can be observed that indicate the character of wave movement through the volume of the parallelepiped, which agreed with theoretical conclusions [11,17].

Based on the previous analysis, it can be claimed that multi-parameter Fourier transform reliably enable data analysis of vibrations, which can have a practical applications in examining vibrations in motor vehicles, because the complex data on registered vibrations on any of the motor vehicle systems can be subjected to multi-parameter frequency analysis.

Calculated spectrum magnitude and phase angles, using multi-parameter inverse Fourier transform, enable the generation of identical vibrations both in the laboratory and under operating conditions [20].

It should be noted that in Fourier transform, there are no explicit procedures for calculating errors in spectrum calculation for multiple variables, as in the case of 1D Fourier transformation [10]. Bearing this in mind, and given that this study aims to illustrate the possibility of applying multi-parameter frequency analysis in examining vibrations in motor vehicles, statistical error analysis was not specifically considered.

Therefore, the choice of integration parameters of partial differential equations was made based on the possibility of implementing 2D, 3D, and 4D Fourier transform algorithms on computers of medium configuration. Finally, it should be noted that these transformations require high-performance computer systems both in the realization phase of exploitation research and in the phase of calculating inverse Fourier transformations for laboratory purposes.

CONCLUSIONS

Based on the performed research, it can be concluded that multi-parameter Fourier transforms reliably enable the analysis of experimental data on vibrations of the continuously bodies, indicating the possibility of their application in motor vehicles.

Calculated spectrum magnitude and phase angles, using inverse Fourier transform, enable the generation of identical vibrations both in the laboratory and under operating conditions.

It should be noted that the Fourier transform algorithms used require high-performance computer systems both in the realization phase of exploitation research and in the phase of calculating inverse Fourier transformations for laboratory purposes.

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