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DIMENSIONING OF THE TORSION BAR OF THE SUSPENSION SYSTEM IN THE PHASE OF THE CONCEPTUAL DESIGN OF A PASSENGER VEHICLE FROM THE PERSPECTIVE OF MINIMAL VIBRATIONS

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RESEARCH ARTICLE

ABSTRACT: Torsion bars are components of the suspension system in motor vehicles, widely used across various vehicle categories due to their simplicity, durability, and compactness. They are most commonly found in off-road vehicles, SUVs, pickup trucks, and light commercial vehicles, where they are valued for their ability to withstand heavy loads and demanding terrain conditions. Torsion bars were also frequently used in passenger cars, especially by European and Japanese manufacturers, in the mid-20th century.

In the conceptual design phase, most vehicle parameters are unknown. This paper presents a procedure for the automated selection of the torsion bar radius using the "Simulated Annealing method". The radius is chosen to minimize the torsional vibrations of the bar, and its verification is carried out by calculating the safety factor under rigorous operating conditions.

KEY WORDS: *vehicle, suspension system, torsion bar, simulated annealing method, vibrations*

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DIMENZIONISANJE TORZIONE ŠIPKE SESTEMA ELASTIČNOG OSLANJANJA U KONCEPTUALNOJ FAZI PROJEKTA PUTNIČKOG VOZILA IZ UGLA MINIMALNIH VIBRACIJA

REZIME: Torzioni štapovi predstavljaju element sistema oslanjanja motornih vozila, koji je našao široku primenu u različitim kategorijama vozila, zahvaljujući jednostavnosti, izdržljivosti i kompaktnosti. Oni se najčešće koriste kod terenskih vozila, SUV modela, pikapova i lakih komercijalnih vozila, gde su poznati po svojoj sposobnosti da izdrže velika opterećenja i zahtevne terenske uslove. Takođe, u sredinom prošlog veka su često korišćeni kod putničkih automobila, posebno evropskih i japanskih proizvođača.

U toku izrade idejnog projekta većina parametara vozila nije poznata. Zbog toga je u ovom radu razvijen postupak za automatizovan izbor poluprečnika torzionog štapa, primenom metode poznatom pd nazivom “Simulisana metoda kaljenja” . Izbor poluprečnika je vršen iz uslova minimizacije torzionih vibracija štapa, a provera je realizovana izračunavanjem stepena sigurnosti u rigoroznim eksploatacionim uslovima.

KLJUČNE REČI: *Vozilo, Sistem za oslanjanje, Torzioni štap, Simulisana metoda kaljenja, vibracije*

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INTRODUCTION

Torsion bars are important components of the suspension system in motor vehicles, widely used in various vehicle categories due to their simplicity, durability, and compactness. These elastic elements work on the principle of twisting around their longitudinal axis, absorbing the loads that occur during driving, thus contributing to the vehicle's stability and comfort. Torsion bars are most commonly found in off-road vehicles, SUVs, pickup trucks, and light commercial vehicles, where they are known for their ability to withstand heavy loads and demanding terrain conditions. They were also used in passenger cars, particularly by European and Japanese manufacturers [1,2].

The application of torsion bars extends beyond civilian vehicles. They are also crucial in military vehicles, such as tanks and armored personnel carriers, where their robustness and ability to withstand extreme loads are critical. Furthermore, they offer greater freedom in chassis design, as they occupy less space compared to traditional spring suspension systems [1,2].

The main advantages of torsion bars include compactness, durability, and easy maintenance, while their disadvantages are the limited ability to adjust ride characteristics and lower comfort compared to modern independent suspension systems.

Despite these drawbacks, torsion bars still play an essential role in specific types of vehicles that require reliability and the ability to withstand harsh operating conditions. Recently, they have been used in combination with elements of active suspension systems [1].

The torsion bar in a vehicle is primarily subjected to torsion [3]. However, during operation, there may also be minimal bending due to other forces (such as weight or lateral loads). These forces are generally small compared to the dominant torsional load.

Torsion bars have been used in vehicles such as the Toyota Land Cruiser, Nissan Patrol, Jeep Cherokee, Chevrolet Silverado, Ford Ranger, and Chrysler Dodge Dart 1, among others [1,2].

For illustration, Figure 1. shows the torsion bar in the suspension system of the Chrysler Dodge Dart 1 from the 1950s-60s. As its function is detailed in [2,3], it will not be further discussed here.

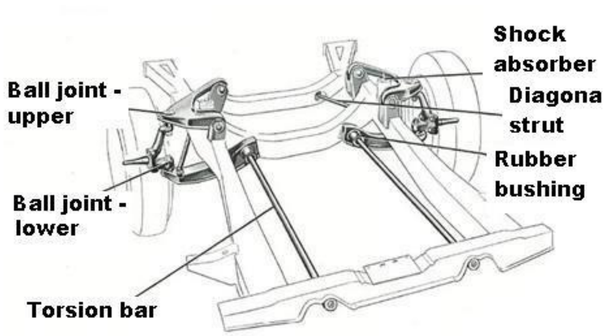

Figure 1. Concept of the suspension system in the Chrysler Dodge Dart 1, 1973

For further reference, Figure 2 shows the isolated torsion bar with the loads it is exposed to [3].

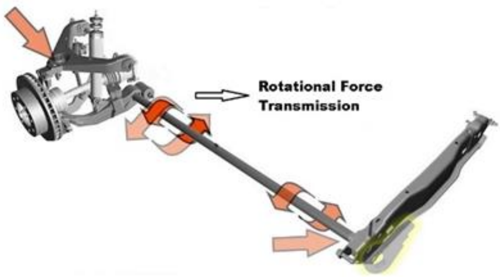

Figure 2. Loads on the torsion bar

It is evident that due to the vertical oscillations of the wheel, which are transmitted to the front end of the torsion bar, and considering that the rear end is clamped (attached to the body), the bar is subjected to twisting. This will be explained further below.

Since the parameters of the motor vehicle, and consequently the torsion bars, are unknown in the conceptual design phase, an attempt will be made to develop a method for selecting the torsion bar's radius, as the length is generally determined by the vehicle's design, usually 1-1.5, meters [1]. Specifically, the dimensioning will be based on minimizing its torsional vibrations.

1. METHOD

It was deemed appropriate to idealize the torsion bar for the study of forced torsional vibrations and to consider it as a homogeneous bar of length 1024, mm, with an unknown radius. The bar undergoes torsional vibrations due to the disturbing torque, which in this case originates from the spatial motion of the wheels and the vehicle body. With this in mind, further discussion will focus on modeling the torsional vibrations of the torsion bar.

1.1 Torsion bar model

In defining the model that describes the forced torsional vibrations of the elastic torsion bar, the following assumptions were made:

- the bar is homogeneous and has a constant diameter, and
- the effect of bending moments is neglected.

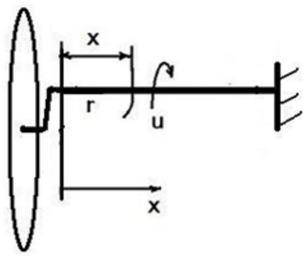

Figure 3. Torsion bar model

The partial differential equation that describes the forced torsional vibrations of the elastic bar is detailed in [4,5]. Its final form will be presented here:

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial x^2} + M(x,t) \,, \qquad (1)$$

where:

- *u(x,t)*- torsional vibrations of the bar,
- *x* - coordinate along the length of the bar,
- *M(x,t)* - forcing torque transmitted from the wheel to the bar,
- *G* - shear modulus of the bar material,
- ρ - density of the torsion bar material, and
- *t*- time.

The forcing torque acting on the torsion bar can be obtained through experimental investigations or dynamic simulation [6-8]. In the absence of other data, it was deemed appropriate to model it using the following function:

$$M_t = A\left[a_1 \sin\left(2\pi f_b t\right) + a_2 \sin\left(2\pi f_e t\right) + a_3 \sin\left(2\pi f_b t\right)\right], \qquad (2)$$

where:

- the amplitude of the forced vibrations acting on the front end of the torsion bar,
- a1,a2,a3 – the amplitudes that take into account the impact of resonant vibrations from the body, engine, and wheels, and
- t- time.

$$M_t = GI_o \frac{\partial u\left(x,t\right)}{\partial x} \qquad (3)$$

where:

•I_0- polar moment of inertia of the cross-section of the bar, expressed by [10]:

$$I_o = \frac{\pi r^4}{2} \,,$$

•*r* - radius of the torsion bar.

Defining the boundary conditions in the analysis of elastic body vibrations generally represents an idealization of the real situation. In this specific case, it was assumed that one end is quasi-free (subjected to a torque due to the variable dynamic ground reaction), while the other end is clamped. Additionally, it was assumed that the torsional vibrations and their velocities are zero at the initial moment, i.e.:

$$\begin{aligned} & u(L,t)=0 \\ & GI_o \frac{\partial u(0,t)}{\partial x}=M_t \\ & u(x,0)=0 \\ & u`(x,0)=0 \end{aligned} \tag{4}$$

The partial differential equation (1), along with the boundary and initial conditions (4), cannot be solved in its closed form. Therefore, it was solved numerically [11] using the finite difference method. Since this procedure is well-known from [11], it will not be further discussed here.

Using the mentioned method, the author developed a program in Pascal and numerically solved the partial differential equation (1), with the excitation function (2) and boundary and initial conditions (4). It should be noted that in the case of numerically solving partial differential equations, it is sometimes necessary to introduce additional boundary and initial conditions [9].

In this specific case, torsional vibrations will be used to dimension the torsion bar using optimization methods, which will be discussed in the following text.

2. OPTIMIZATION METHOD

There are numerous methods for nonlinear programming in the literature [12], which are detailed in [12], and some of them have been used for the optimization of oscillatory parameters of motor vehicles and their systems [13-15]. It should be noted that these methods predominantly discover local minima of the objective function, and the global minimum can be discovered by applying multiple starting values for the optimizing parameters.

One of the newer methods, called the "Simulated Annealing Method" (hereinafter referred to as the "Optimization Annealing Method" – OAM), is more recent [16]. Simulated annealing is an optimization method inspired by the process of thermal hardening of materials. In this process, energy represents the objective function. The goal of the simulated annealing method is to gradually bring the system to a state with the lowest possible "energy" – in other words, to find the global minimum of the objective function.

Temperature is an abstract parameter that controls the probability of accepting worse solutions during optimization. At high temperatures (at the beginning of the process), the algorithm is "gentler" and has a higher probability of accepting worse solutions. As the temperature decreases, the algorithm becomes "stiffer" and only accepts solutions that significantly reduce energy, ensuring convergence toward the global minimum of the objective function. Given that the method is detailed in [16], its structure will not be further discussed here.

In this paper, an attempt was made to use the aforementioned method for dimensioning the torsion bar of the vehicle's suspension system. For this purpose, a software was implemented in Pascal, and its block diagram is shown in Figure 4.

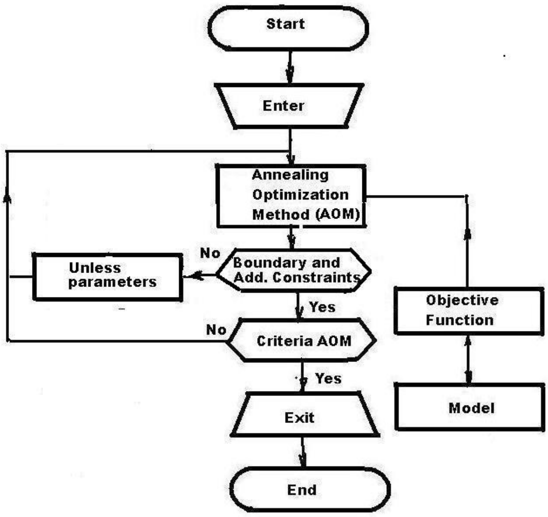

Figure 4. Block Diagram of the software for dimensioning the torsion bar using the simulated annealing method

From the figure, the concept of the optimization process itself can be traced. Initially, the necessary data required for the dynamic simulation of the torsional vibrations of the bar are entered, including data on the number of integration points, integration steps, and initial values (radius) of the bar. When the Optimization Annealing Method (OAM) is called, a subprogram for calculating the objective function is automatically triggered, which calls the torsion bar model. The boundary values of the optimizing parameter and any additional relationships between the optimizing parameters (if applicable) are then checked. If these conditions are not met, the data are considered unacceptable, and the process returns to OAM. If the criteria are satisfied, the stopping criterion for the iterative process is checked. When the stopping criterion is met, the optimization process concludes. If not, the process returns to the simulated annealing method.

To minimize the torsional vibrations of the bar, the following objective function was used:

$$Z = u_{RMS} \tag{5}$$

where:

- u_{RMS} - RMS values of the forced vibrations of the bar obtained by solving the partial differential equation (1).

The RMS of the vibrations is calculated using the following expression:

$$a_{RMS}^2 = \frac{1}{n_x n_t} \sum_{i=1}^{n_x} \sum_{j=1}^{n_t} u(i,j)^2 , \tag{6}$$

where:

- u(i,j) - torsional vibrations of the bar,
- n_x - number of points along the x-axis, and
- n_t - number of points along the t-axis.

The optimization process was carried out with the introduced boundary values for the radius of the bar:

$$5 < r < 25, \tag{7}$$

To define the integration steps of the partial differential equation (1), the Courant-Friedrichs-Lewy (CFL) method was used, which ensures numerical stability during its solution. More precisely, it ensures a good connection between the temporal and spatial coordinates of the system [1, 17, 18]. For one-dimensional vibrations, it is defined as:

$$CFL = \sqrt{\frac{G}{\rho}} \frac{\Delta t}{\Delta x} \leq 1, \tag{8}$$

where:

- Δx and Δt are the spatial and temporal increments, respectively.

Using equation (8), for an integration step of 2, mm along the x-axis, the time step was calculated to be 0.00002, s. This data ensures reliable frequency analysis of the data along the "x" axis from 0.00097 to 0.25, 1/mm, and the "t" axis from 5.10 to $5e^{+4}$,Hz [19]. The upper value of the time frequency allows for realistic vibration analysis of the bar, as it covers the area of vibration occurrence in solid bodies.

The data used for the dynamic simulation are presented in Table 1. (for illustrative purposes, corresponding to a heavier passenger vehicle):

Table 1. Data Necessary for Dynamic Simulation

Parameter	Value
Mass of the loaded vehicle, m, kg	1400
Mass of the wheel, m_w, kg	20
Wheelbase, L, m	2.946
Distance from the center of gravity to the rear axle, b_v, m	1.7
Dynamic wheel radius, r_d, m	0.3
Height of the center of gravity from the ground, h_v, m	0.95
Allowable torsional stress, τ, MPa	300

It should be noted that the allowable torsional stress for steel springs ranges between 400-600, MPa [19], but a lower value has been adopted here to ensure that torsion bars are used within the material's elastic limits [1].

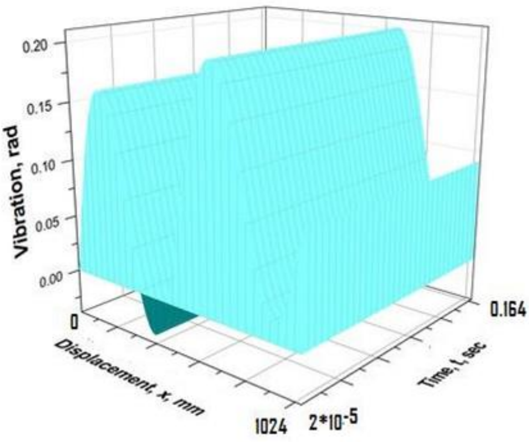

Figure 5. Torsional vibrations of the torsion bar for the optimal radius

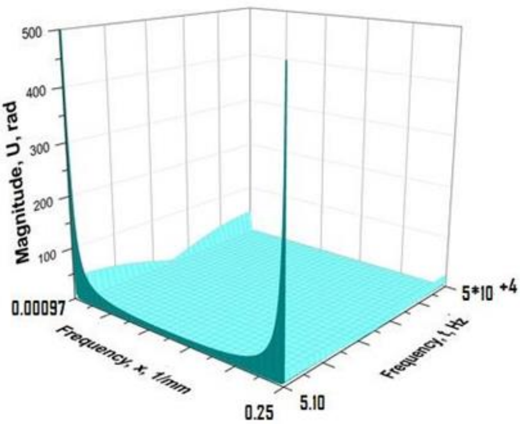

Figure 6. Vibration magnitude of the torsion bar for the optimal radius

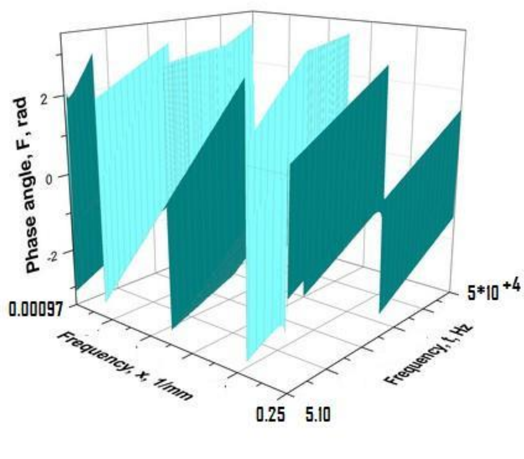

Figure 7. Phase angles of the torsion bar for the optimal radius

The optimization was performed on a Pentium 4 computer (Intel 2.4 GHz, 9 GB RAM), and the iterative process was automatically stopped when the difference between two successive values of the objective function reached $1e^{-10}$. The optimization time was about 10 minutes, and the calculated radius of the torsion bar was r_{opt} = 16.1752744, mm, with a minimum objective function value of 0.0005340, rad.

Based on the calculated radius, a standard size for the torsion bar diameter should be selected, but this will not be discussed further here.

3. DATA ANALYSIS

The calculated radius of the torsion bar was determined based on the minimization of its torsional vibrations. It was deemed appropriate to analyze its safety factor under rigorous operating conditions.

As is well known, it is defined by the expression:

$$\nu = \frac{\tau_a}{\tau}, \qquad (9)$$

where:

- τ_a_- allowed - allowable torsional stress (given in Table 1), and
- τ_- actual - actual torsional stress in the torsion bar.

It is well known that the torsion bar's loading check is performed under conditions of the vehicle's curved motion. For passenger vehicles, the maximum centrifugal force is adopted as [21]:

$$F_c = 0.4 m_s g, \qquad (10)$$

where:

- m_s - supported vehicle mass, and
- g - gravitational acceleration.

The centrifugal force transferred to the front suspension system is given by the expression [8]:

$$F_{cf} = F_c \frac{b_v}{L_v}, \qquad (11)$$

where:

- b_v - coordinate of the vehicle's center of gravity from the rear axle, and
- L_v - wheelbase (Table 1).

The forcing torque applied to one torsion bar is expressed by the formula:

$$M_t = F_{cf}\left(h_t - r_d\right), \qquad (12)$$

where:

- h_t - height of the center of gravity of the supported vehicle mass from the ground, and
- r_d - dynamic radius of the tire.

Torsional stress is given by the expression:

$$\tau = \frac{2M_t}{\pi r^3}, \tag{13}$$

Bearing in mind the expression (9-13) and the allowable torsional loading (Table 1), the safety factor was calculated as 5.615. Since for motor vehicles, the typical safety factor ranges between 2-3 (except in rigorous cases), the safety factor meets the recommendations and allows for a reduction in the radius, though with an increase in vibration levels. Given that torsion bars are exposed to rigorous operating conditions and considering material fatigue, it is deemed appropriate to adopt the optimal radius of the bar for further design.

It was also considered appropriate to further analyze the vibrations for the optimal bar diameter. In this context, equation (1) was solved, and the results are shown in Figure 5. From the figure, it is evident that the vibrations propagate in waves along the length of the torsion bar, which aligns with the findings in [4, 5].

Based on the data obtained from the dynamic simulation, shown in Figure 5, a frequency analysis was performed using the so-called “2D” Fourier transform. This was done using software [22], and the results are shown in Figures 6 and 7.

Analysis of Figures 6 and 7 shows that the magnitudes and phase angles depend on the frequency along the "x" and "t" axes. It is important to note that the magnitude of the spectrum allows for the analysis of resonant frequencies of the bar and comparison with the resonances of the vehicle system, which should not coincide [23].

It should be noted that with “2D” Fourier transforms, there is no procedure for calculating the error in spectral analysis, as is the case with one-dimensional problems [19]. Therefore, such an analysis was not performed in this work.

Additionally, the calculated radius was compared with those found in actual vehicles. According to [1], the diameters range from 20-30, mm (for the Dodge Dart 1, 22.6, mm).

Since the optimal calculated radius is 16.1752744,mm, it is evident that the developed method gives slightly higher values than those found in actual vehicles, but this data is useful as a guideline in the conceptual design phase of the vehicle.

4. CONCLUSION

Based on the conducted research, it can be concluded that the developed method for dimensioning torsion bars in the suspension system can serve as a guideline during the conceptual design phase of a vehicle. The safety factor calculated based on the optimal radius of the torsion bar in the suspension system is satisfactory. The final decision regarding the dimensions of the torsion bars should be made based on an analysis of the resonances of the vehicle and torsion bars, as well as their real loading conditions. The simulated annealing method used here for optimization is stable and applicable in the conceptual design phase of the vehicle, as it reliably detects the global minimum of the objective function.Moreover, continuous advancements in technology, particularly those related to Industry 4.0, such as the integration of smart grids, 5G, and real-time monitoring systems, are pivotal in enhancing the safety and reliability of EV charging and operation. These technologies can help predict and manage risks associated with electrical hazards,

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