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## ANALYSIS OF CRANKSHAFT TORSIONAL OSCILLATION DUMPER FOR ENGINE V-46-6

Marko Nenadović<sup>1\*</sup>, Dragan Knežević<sup>2</sup>, Željko Bulatović<sup>3</sup>

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RESEARCH ARTICLE

**ABSTRACT:** The crankshaft of an internal combustion engine represents an elastic system that is susceptible to deformations due to the effects of external forces and moments. The forces and the resulting torsional moments that stress the crankshaft during engine operation are periodic in nature, which causes torsional oscillations of the crankshaft. As a result of torsional oscillations, the crankshaft undergoes additional stresses. In some cases, these stresses can far exceed permissible values, which is why they must be checked during the engine design phase. Exceptionally intense torsional oscillations are particularly common in long shafts, as well as those shafts that are subjected to periodically varying torsional moments of extremely large amplitude. This paper will analyze the process of torsional oscillation of the crankshaft of a special-purpose engine, characterized by both of the mentioned parameters, which, among others, negatively affect the additional stresses on the crankshaft due to torsional oscillations. Additionally, the paper will analyze the methods by which it is possible to reduce the amplitudes of oscillations if necessary.

**KEY WORDS:** *Crankshaft, torsional oscillations, torsional vibration dumper*

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<sup>1</sup>Marko Nenadović, Military Technical Institute, Ratka Resanovica 1, Belgrade, Serbia,

mare.nenadovic92@gmail.com,  <https://orcid.org/0009-0004-0172-3216>, (\*Corresponding author)

<sup>2</sup>Dragan Knežević, University of Belgrade Faculty of Mechanical Engineering Department of Internal Combustion Engines, Kraljice Marije 16 11000 Belgrade, Serbia, dknezevic@mas.bg.ac.rs ,

 <https://orcid.org/0000-0003-0403-3101>

<sup>3</sup>Željko Bulatović, Military Technical Institute, Ratka Resanovica 1, Belgrade, Serbia,

zetonbulat@gmail.com,  <https://orcid.org/0009-0009-2339-1933>

## **ANALIZA TORZIONOG OSCILACIONOG PRIGUŠAČA KOLENASTOG VRATILA ZA MOTOR V-46-6**

**REZIME:** Radilica motora sa unutrašnjim sagorevanjem predstavlja elastični sistem koji je podložan deformacijama usled delovanja spoljašnjih sila i momenata. Sile i rezultujući torzioni momenti koji opterećuju radilicu tokom rada motora su periodične prirode, što uzrokuje torzione oscilacije radilice. Kao rezultat torzionih oscilacija, radilica je podvrgnuta dodatnim napreznjima. U nekim slučajevima, ova napreznja mogu daleko premašiti dozvoljene vrednosti, zbog čega se moraju proveriti tokom faze projektovanja motora. Izuzetno intenzivne torzione oscilacije su posebno česte kod dugih vratila, kao i kod onih vratila koja su izložena periodično promenljivim torzionim momentima izuzetno velike amplitude. U ovom radu će se analizirati proces torzionog oscilovanja radilice motora specijalne namene, koji karakterišu oba pomenuta parametra, a koji, između ostalog, negativno utiču na dodatna napreznja na radilici usled torzionih oscilacija. Pored toga, u radu će se analizirati metode kojima je moguće smanjiti amplitude oscilacija ako je potrebno.

**KLJUČNE REČI:** *Radilica, torzione oscilacije, prigušivač torzionih vibracija*

# ANALYSIS OF CRANKSHAFT TORSIONAL OSCILLATION DUMPER FOR ENGINE V-46-6

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## INTRODUCTION

The crankshaft is one of the most structurally complex, heavily loaded, critical, and expensive components of an internal combustion engine, which is why special attention is given to its design. During engine operation, in addition to other forces, the crankshaft is subjected to periodically varying torsional moments, which, due to their periodic nature, lead to torsional oscillations of the crankshaft, causing additional stresses.

The crankshaft, like any other elastic element, has its own natural frequencies of oscillation. On the other hand, the torsional excitation moments that load the crankshaft, being periodic in nature, have their own frequency, which represents the excitation frequency and depends on the crankshaft's rotational speed. Consequently, it is possible that at a certain rotational speed, the excitation frequency may coincide with one of the natural frequencies of the crankshaft's oscillations, leading to resonant oscillations. During resonant oscillations, the oscillation amplitudes can increase drastically (compared to non-resonant regimes), causing the crankshaft's stresses to exceed permissible values, potentially leading to the failure of the crankshaft itself. Additionally, crankshaft operation at resonant regimes is accompanied by increased noise and unpleasant vibrations of the entire engine [1,2].

If the calculation of torsional oscillations reveals that the crankshaft stresses exceed permissible limits, certain measures must be taken to reduce these stresses to within acceptable values. To reduce stresses due to torsional oscillations, the rotational speeds at which the stresses caused by torsional oscillations are excessive can be marked on the tachometer, and these rotational speeds must be avoided during engine operation. This method of protecting the crankshaft from resonant oscillations is limited to applications where the engine can operate at a certain rotational speed for a very short time (e.g., only during acceleration), which is the case with marine engines, and therefore cannot be applied in other engine applications (e.g., vehicle engines). Additionally, to reduce stresses caused by torsional oscillations, certain design modifications can be made to the crankshaft itself, such as changing the moments of inertia and torsional stiffness, which leads to a change in the natural frequency of oscillation, and consequently a change in the amplitudes during resonant oscillation. This method of stress reduction is rarely used due to its complexity and extremely high material costs. The most commonly used method for reducing additional stresses due to torsional oscillation is the installation of torsional vibration dampers on the crankshaft itself. Installing a damper does not require any structural modifications to the engine, making this procedure cost-effective.

## 1 EQUIVALENT TORSIONAL OSCILLATORY SYSTEM

In real torsional oscillatory systems that contain elements of complex shapes, such as the torsional oscillatory system of an internal combustion engine, it is particularly challenging to perform any calculations. To facilitate the calculation, certain simplifications are introduced. All simplifications are based on replacing the real shapes of the shafts, which perform rotational movements, with light shafts (of negligible mass) with specific torsional stiffness and concentrated masses with certain moments of inertia. The values of the torsional stiffness of the light shafts and the moments of inertia of the concentrated masses

are determined based on the condition of equality of the kinetic and potential energy of the real and simplified oscillatory system. From the condition of energy equality between the real and simplified system, it follows that the concentrated masses of the simplified system must have the same moments of inertia, while the light shafts must have the same torsional stiffness as the parts of the real system they represent. The simplified oscillatory system thus obtained is called an equivalent torsional oscillatory system [2,3,4].

When the internal combustion engine is considered as a torsional oscillatory system or part of a more complex oscillatory system (engine and transmission), practical reasons lead to focusing on the crankshaft, which, due to its specific geometry and constant excitation during operation, is the main source of torsional oscillations. The other subsystems of the engine are then reduced relative to the crankshaft, adhering to the aforementioned condition of energy equality between the real and equivalent torsional oscillatory system [2].

The moments of inertia of the concentrated masses, as well as the torsional stiffness of the sections (light shafts,  $I_s$  at Figure 1) of the equivalent torsional oscillatory system engine-dynamometer, for the “V-46-6” engine, which is the subject of this study, are shown in Table 1. The procedure for determining these parameters is described in detail in reference [2]. The appearance of such an equivalent torsional oscillatory system can be seen in Figure 1, while the real torsional oscillatory system can be seen in Figure 2.

In addition to the moments of inertia of the concentrated masses and the torsional stiffness of individual sections, for the complete definition of the equivalent torsional oscillatory system, it is necessary to determine the damping (internal and external), as well as the excitation torsional moments that load the crankshaft.

Due to the small number of studies addressing the issue of damping in the torsional oscillatory system of an engine, determining the damping represents the most complex part of defining the equivalent torsional oscillatory system. Here, the damping was calculated according to recommendations found in literature from Eastern countries. The damping thus calculated is external by its nature, taking into account the influence of internal damping in the material.

**Table 1.** Parameters of the equivalent torsional oscillatory system [2]

	Moment of inertia $J$ ( $\text{kgm}^2$ )	Torsional stiffness $c$ ( $\text{Nm/rad}$ )	Dumping coefficient $k$ ( $\text{Nms/rad}$ )
Mass 1	0.010441		0
Light shaft 1		$2.074 \times 10^6$	
Mass 2	0.172104		8.1518
Light shaft 2		$2.188 \times 10^6$	
Mass 3	0.171664		8.1518
Light shaft 3		$2.188 \times 10^6$	
Mass 4	0.171664		8.1518
Light shaft 4		$2.188 \times 10^6$	
Mass 5	0.171664		8.1518
Light shaft 5		$2.188 \times 10^6$	

Mass 6	0.171664		8.1518
Light shaft 6		$2.188 \times 10^6$	
Mass 7	0.171664		8.1518
Light shaft 7		$8.885 \times 10^6$	
Mass 8	0.278054		0
Light shaft 8		90000	
Mass 9	1.755389		0

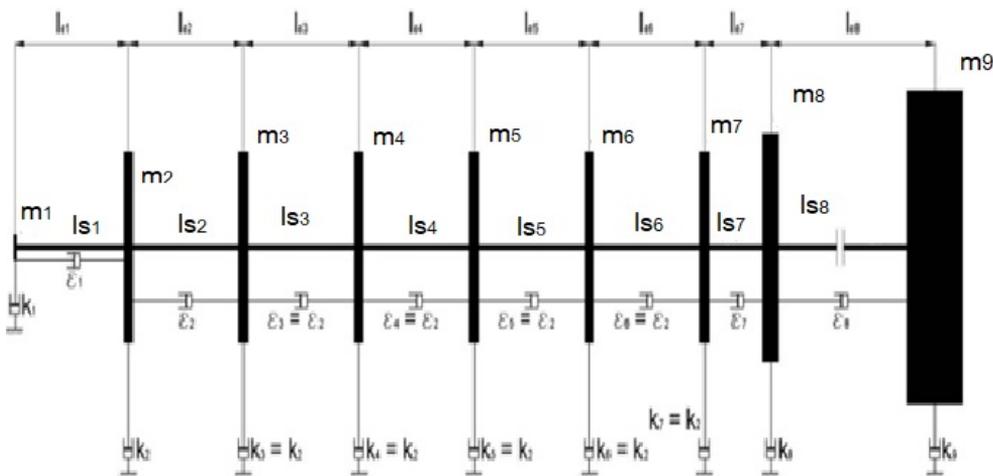


Figure 1 Equivalent torsional oscillatory system engine-dynamometer [2]

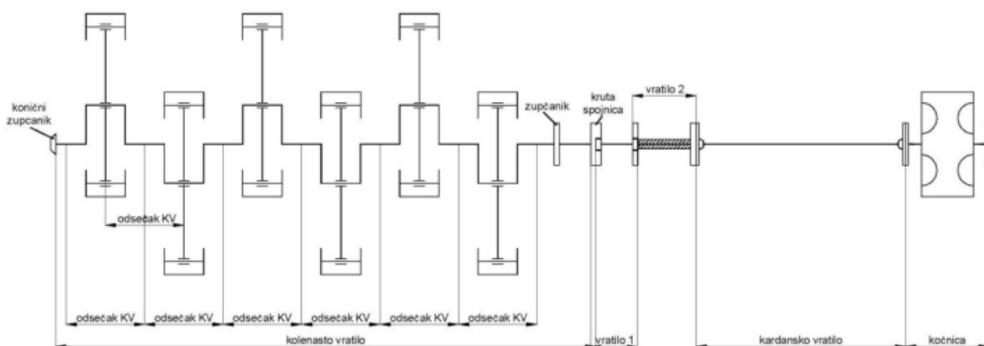


Figure 2 Real torsional oscillatory system [2]

The excitation moments that load the crankshaft are the result of the action of gas forces and the forces of inertia of the linearly oscillating masses. The inertia forces of the rotating masses do not affect torsional oscillations.

To determine the excitation moment, it is necessary to know the pressure curve in the engine cylinder as a function of the crankshaft angle [5]. By indicating the pressure in the first left and third right cylinders, for the regime of maximum torque, the pressure curves as a function of the rotation angle were obtained and are shown in Figure 3.

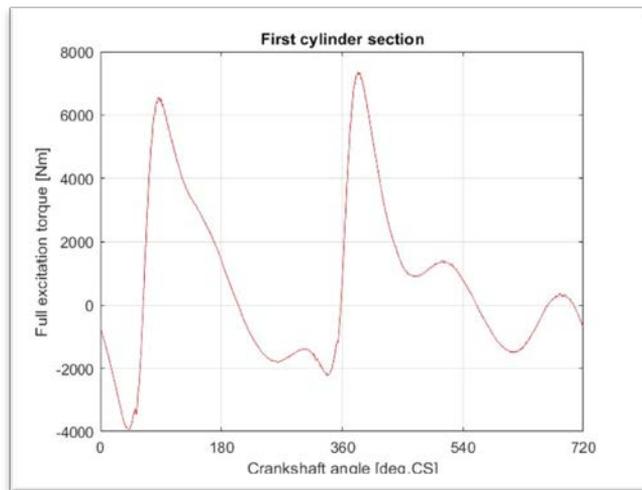
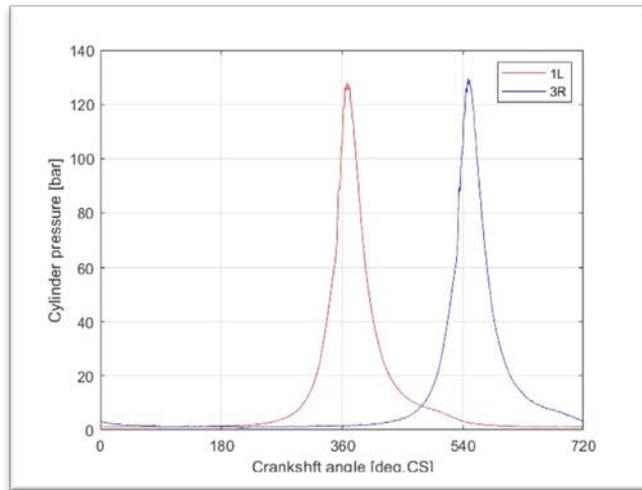


Figure 3 Pressure curve in the first left and third right cylinders as a function of the crankshaft rotation angle

Figure 4 Total excitation moment curve as a function of the crankshaft angle, at 2000 rpm

When the pressure curves in the cylinder are known, along with the kinematic parameters and masses of the piston mechanism, the total excitation torsional moment (combined effect of the gas force moment and the inertial force moment of the linearly oscillating masses) can be easily obtained. It should be noted that the decomposition of forces in the piston mechanism, for the engine in question, cannot be performed in the same way for the cylinders in the left and right banks. This is because the piston mechanism of one section (opposite cylinders in different banks) of the cylinders is designed with a main (left bank) and an auxiliary connecting rod (right bank), known as a compound piston mechanism. A detailed description of the procedure for decomposing the forces of the compound piston mechanism can be found in reference [2,3]. By applying this procedure, the total excitation moment curve as a function of the crankshaft rotation angle for the first cylinder section was obtained, and it is shown in Figure 4. Assuming that the working cycles in each left and each right cylinder are identical, the excitation moments in the other cylinder sections are

identical to those in the first section but phase-shifted by the firing angle between the specific section and the first section.

The analysis of torsional oscillations of the crankshaft with excitation as shown in Figure 4 is extremely complex because the total excitation moment is not a simple harmonic function of the crankshaft rotation angle. Therefore, to simplify the calculation, the total excitation moment is expanded into a Fourier trigonometric series. By expanding the total excitation moment into a Fourier series, the excitation moment is obtained as the sum of simple harmonic functions, which significantly simplifies the calculation [6]. In the calculation, each harmonic of the excitation moment must be analyzed individually, and after such an analysis, the cumulative effect of all harmonics may occur. The expansion of a function defined at discrete points, such as the excitation moment function, into a Fourier series is described in references [2,3]. In Figure 5, the amplitude of each harmonic of the Fourier series for the defined excitation moment can be seen.

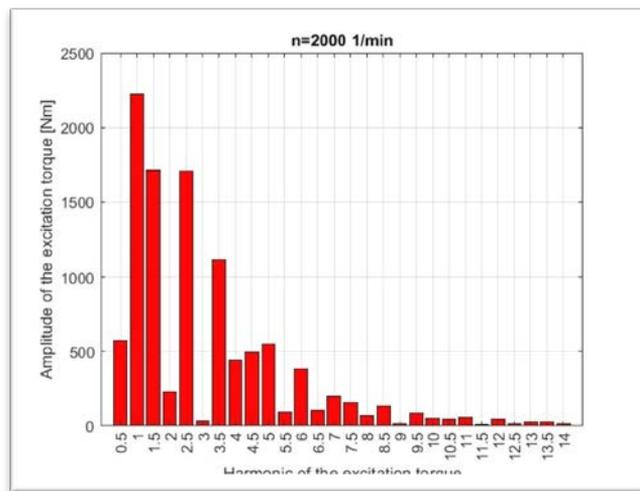


Figure 5 Amplitudes of all harmonics of the total excitation moment

## 2 NATURAL FREQUENCIES OF OSCILLATION

The natural frequencies of oscillation of the equivalent torsional oscillatory system of an internal combustion engine are determined under the assumption that the oscillations are free and undamped, meaning that the effects of damping and excitation moments on the oscillation process are neglected. This assumption has proven to be entirely justified in systems where there is no significantly large source of damping [7].

For oscillatory systems with more than two degrees of freedom, as is the case here, it is quite complicated to analytically determine the natural frequencies of oscillation. Therefore, in such systems, the natural frequencies of oscillation are typically determined numerically, with Holzer numerical method being the most commonly used. A detailed description of the application of Holzer numerical method is provided in references [2,3].

By applying Holzer method to the equivalent torsional oscillatory system from this study, the following values were obtained for the first three natural frequencies: 337.67 Hz – first natural frequency, 1541.6 Hz – second natural frequency, 3145.91 Hz – third natural frequency. Holzer method can also be used to determine the other natural frequencies (in

this particular case, there are eight in total), but they are not of interest for analysis as they have high values, and the system will never be in resonance at their oscillation modes. The relative amplitudes for the first three modes of oscillation are shown in Figures 6, 7, and 8.

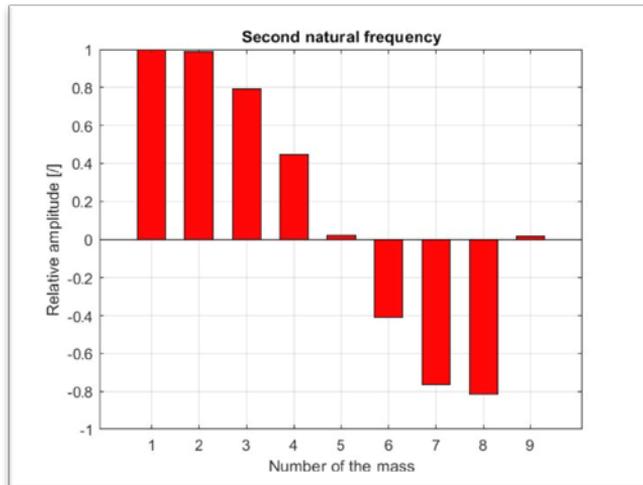
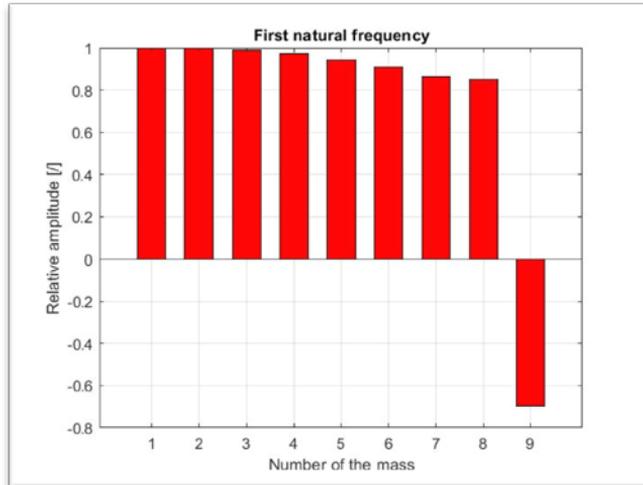


Figure 6 Relative amplitudes for the first mode of oscillation

Figure 7 Relative amplitudes for the second mode of oscillation

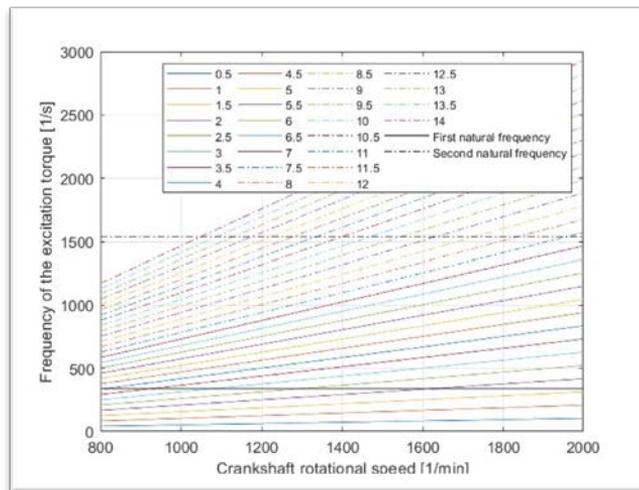
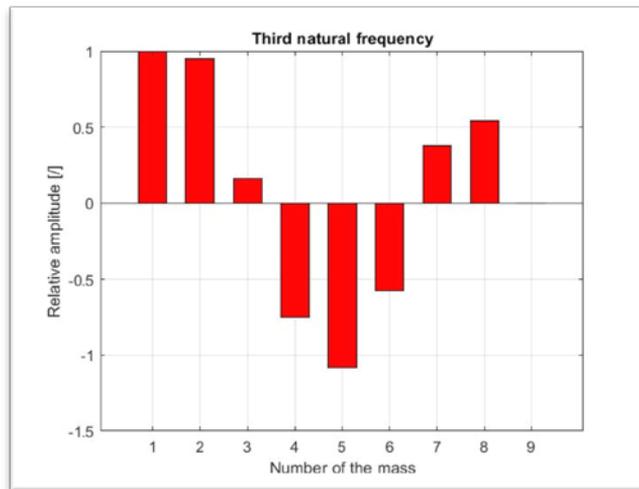


Figure 8 Relative amplitudes for the third mode of oscillation

Figure 9 Campbell diagram

Once the natural frequencies of oscillation are known, it is easy to determine the critical rotational speeds (the rotational speeds at which one of the excitation harmonics is resonant). A graphical representation of the critical rotational speeds is provided in the so-called Campbell diagram, which is shown in Figure 9.

### 3 AMPLITUDES AND ADDITIONAL STRESSES DUE TO TORSIONAL OSCILLATIONS

The amplitudes of all masses in the equivalent torsional oscillatory system are determined by solving a system of differential equations, which is obtained by applying LaGrange equations of the second order for each concentrated mass of the system [4]. The resulting system of differential equations needs to be solved for each harmonic of the excitation moment, at every rotational speed within the operational range. The procedure for solving such a system of differential equations can be found in references [2,3]. By applying such a

mathematical model, the amplitudes of each mass and the stresses in each section as a function of engine speed were obtained, as shown in Figures 10 and 11.

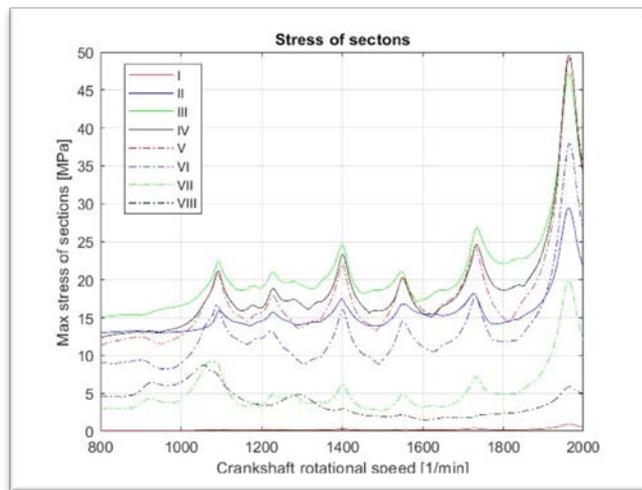
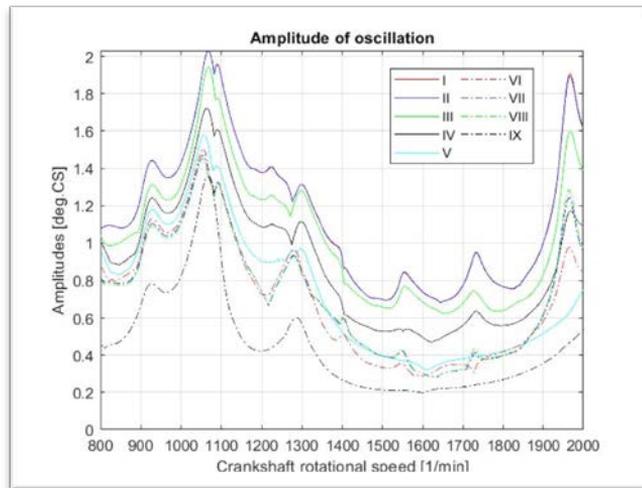


Figure 10 Amplitudes of all masses as a function of engine speed

Figure 11 Stresses in sections as a function of engine speed

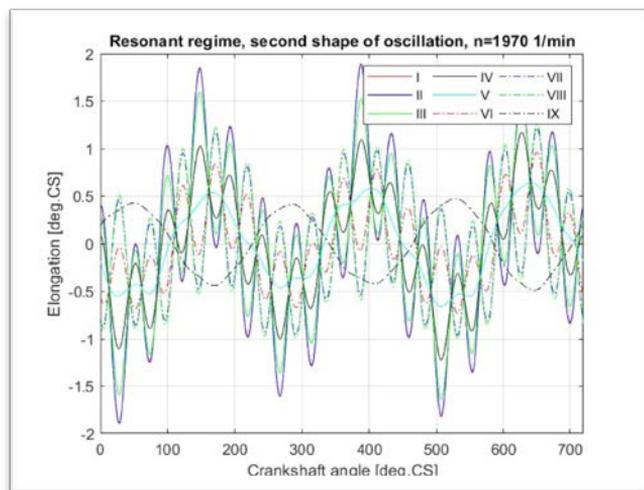
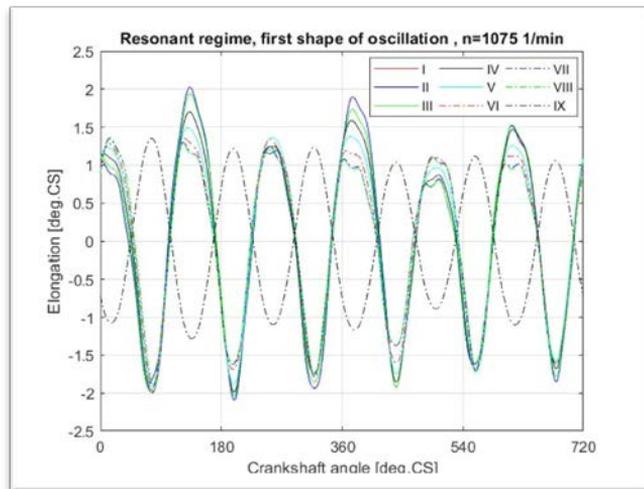


Figure 12 Oscillation flow at 1075 rpm

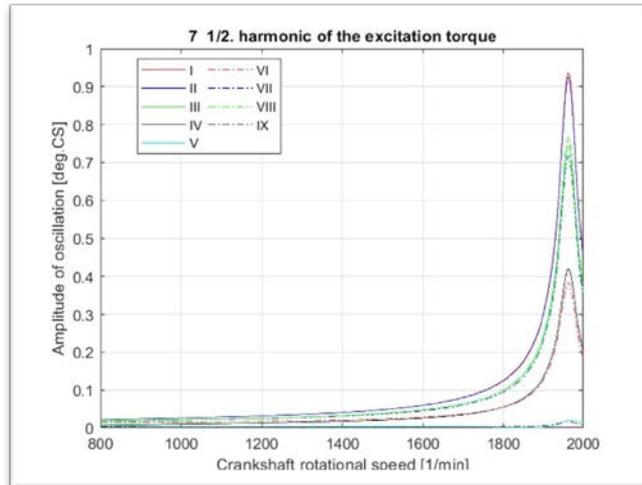
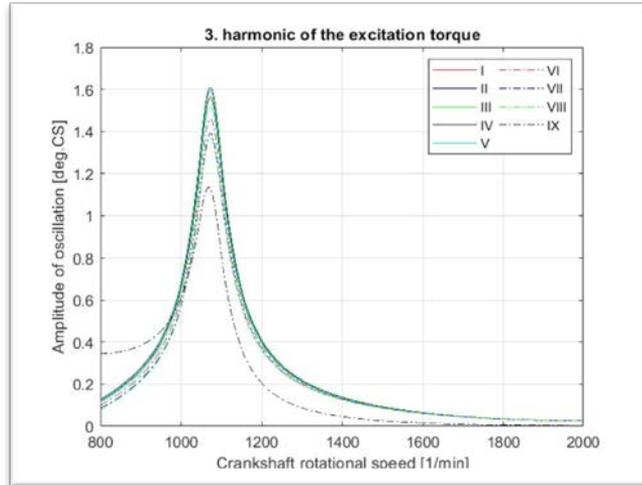
Figure 13 Oscillation flow at 1970 rpm

Figures 12 and 13 show the oscillation flows of each concentrated mass at the rotational speed where the oscillation amplitude, or additional stress, has a maximum value. The maximum oscillation amplitude occurs in the first concentrated mass (conical gear for driving auxiliary devices) of the equivalent system at 1075 rpm. Figure 12 clearly shows that at this speed, the crankshaft oscillates according to the first mode of oscillation.

From Figure 13, it is not possible to reliably determine the mode of oscillation of the crankshaft at 1970 rpm, even though the excitation harmonic of order  $7\frac{1}{2}$  is resonant at this speed in the second mode of oscillation. This can be explained by the fact that at a slightly higher speed of over 2000 rpm, the  $1\frac{1}{2}$  harmonic is resonant in the first mode of oscillation, which influences the oscillation pattern at 1970 rpm. Since the  $1\frac{1}{2}$  and  $7\frac{1}{2}$  harmonics are not resonant in the same mode of oscillation, the mode of oscillation at 1970 rpm due to all the excitation harmonics is not clearly defined.

In the “V-46-6” engine, almost every resonant regime in the first mode of oscillation is accompanied by an excitation harmonic that is resonant in the second mode of oscillation at

nearly the same speed (see Campbell diagram). As a result, the oscillation amplitudes due to the effect of all the excitation harmonics are significantly higher than the amplitudes obtained only from the resonant harmonic at the given critical speed. Figures 14 and 15 show the amplitudes of the concentrated masses due solely to the effect of the 3rd harmonic and the  $7\frac{1}{2}$  harmonic, respectively.



*Figure 14 Amplitudes of all system masses as a function of speed due to effect of the 3rd excitation harmonic*

*Figure 15 Amplitudes of all system masses as a function of speed due to the effect of the  $7\frac{1}{2}$  excitation harmonic*

By comparing the amplitude values obtained solely from the resonant harmonics with those obtained from the effect of all the excitation harmonics, it can be stated that the assertion about the necessity of analyzing torsional oscillations due to all excitation harmonics, in order to obtain a more realistic picture of additional stresses, presented in references [1, 2] is fully justified. For example, the oscillation amplitude of the first mass, due to the effect of all excitation harmonics, is nearly 100% higher at 1970 rpm than that obtained from analyzing only the resonant harmonic at the given speed.

Regarding the additional stresses, they are highest at 1970 rpm, where they exceed the permissible 40 MPa. Therefore, it is necessary to take certain measures to reduce them to acceptable levels. In addition to the high stresses, the exceptionally high oscillation amplitude of the first mass (the bevel gear for driving the engine’s auxiliary devices) throughout the engine’s operating range also poses a problem. This is a particular issue because the conical gear also drives the engine timing mechanism, which disrupts the working fluid exchange process, leading to power loss and reduced engine efficiency. Since the engine in question uses a conventional injection system (pump-pipe-nozzle), these large amplitudes also negatively affect the fuel injection process, as the high-pressure pump is driven by a crankshaft that is oscillating intensely. Due to all these reasons, it is necessary to reduce the level of torsional oscillations in the system, which will be achieved here by applying torsional vibration dampers.

#### 4 TORSIONAL VIBRATION DAMPER

In this study, the simplest model of a torsional vibration damper will be used. This model consists of one concentrated mass representing the entire engine and another concentrated mass representing the inertial mass of the torsional vibration damper. These two concentrated masses are connected by a section with appropriate stiffness, where the internal damping of that section must be considered in the analysis of the torsional damper. The described model of the torsional vibration damper can be seen in Figure 16. In defining this model, the nine masses of the equivalent torsional oscillatory system were reduced to one. In this reduction process, it is necessary to respect the condition of the equality of kinetic and potential energy between the initial system (with nine masses) and the final system (one mass). Although at first glance, representing the entire torsional oscillatory system with only one concentrated mass seems like an extreme simplification, reference [11] shows that applying such a model yields satisfactory results.

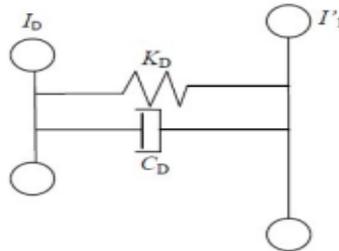


Figure 16 Simple model of a torsional vibration damper [11]

For the system shown in Figure 16, the following two differential equations can be written [11]:

$$\theta_1 \ddot{\vartheta}_1 + c_1 \dot{\vartheta}_1 - c_2(\vartheta_2 - \vartheta_1) - b(\dot{\vartheta}_2 - \dot{\vartheta}_1) = M_1 \cos(\Omega t), \tag{1}$$

$$\theta_2 \ddot{\vartheta}_2 + c_2(\vartheta_2 - \vartheta_1) + b(\dot{\vartheta}_2 - \dot{\vartheta}_1) = 0, \tag{2}$$

where:  $\theta_1$  – moment of inertia of the engine [kgm<sup>2</sup>],  $\theta_2$  – moment of inertia of the damper mass [kgm<sup>2</sup>],  $c_2$  – torsional stiffness of the section between the engine and the damper mass [Nm/rad],  $b$  – internal damping coefficient of the section [Nms/rad],  $\vartheta_1$  – angular displacement of the engine mass [°],  $M_1$  – amplitude of the excitation moment [Nm],  $\Omega$  – frequency of the excitation moment [1/s].

It should be noted here that the torsional stiffness  $c_1$  is determined based on the condition that the natural frequency of oscillation of the mass representing the engine is equal to the excitation frequency of the resonant harmonic at the critical rotational speed.

The solutions to the system of differential equations (1) and (2) are assumed, according to the theory of linear differential equations, to be of the form:

$$\vartheta_i = X_i \cos(\Omega t) + Y_i \sin(\Omega t), \quad (3)$$

When the assumed solution (3) is substituted into equations (1) and (2), a system of four algebraic equations with four unknowns is obtained. This system, in which the unknowns are the oscillation amplitudes of both masses, can be solved using the determinant method. Solving this system yields the following relation for the dynamic amplification factor of the first mass (engine mass):

$$\eta_d = \frac{4\beta^2\psi_1^2 + (\psi_1^2 - \gamma^2)^2}{4\beta^2\psi_1^2(\psi_1^2 - 1 + \mu\psi_1^2)^2 + [\mu\psi_1^2\gamma^2 - (\psi_1^2 - 1)(\psi_1^2 - \gamma^2)]^2} = \frac{M\beta^2 + N}{P\beta^2 + Q}, \quad (4)$$

where:  $\beta = b/2\theta_2\omega_{11}$  – dimensionless damping coefficient [1],  $\omega_{11}$  – natural frequency of oscillation of the engine mass [1/s],  $\mu = \theta_2/\theta_1$ ,  $\gamma = \omega_{22}/\omega_{11}$ ,  $\omega_{22}$  – natural frequency of oscillation of the damper mass [1/s].

For the dynamic amplification factor of the engine mass, expressed by relation (4), to be independent of the damping value, it is necessary for the determinant of the quantities  $M$ ,  $N$ ,  $P$ , and  $Q$  to be equal to zero, i.e., the condition  $MQ - PN = 0$  must be satisfied. Based on this condition, the following biquadratic equation is obtained:

$$\psi_1^4 - 2\frac{1+(1+\mu)\gamma^2}{2+\mu}\psi_1^2 + \frac{2\gamma^2}{2+\mu} = 0, \quad (5)$$

Solving the biquadratic equation (5) yields two values of  $\psi_1^2$  for which the dynamic amplification factor of the engine mass is independent of the internal damping of the section between the engine mass and the damper mass. If it is assumed that the damping is  $\beta = \infty$ , the two values of  $\psi_1^2$  obtained by solving the biquadratic equation (5) give the following values for the dynamic amplification factor of the engine mass:

$$\eta_{d1}^{(1)} = \frac{1}{\left|1 - (1 + \mu)\frac{\Omega_1^2}{\omega_{11}^2}\right|}, \quad (6)$$

$$\eta_{d1}^{(2)} = \frac{1}{\left|1 - (1 + \mu)\frac{\Omega_2^2}{\omega_{11}^2}\right|},$$

It has been shown in references [4,5] that the best value for the dynamic amplification factor of the engine mass is achieved when both dynamic factors in (6) have the same value. By equating them, the most favorable ratio of the natural frequencies of the damper mass and the engine mass can be determined, which should be used in the design of the torsional vibration damper. The most favorable ratio of natural frequencies is:

$$\gamma_{opt} = \frac{\omega_{22}}{\omega_{11}} = \frac{1}{1+\mu}, \quad (7)$$

When the optimal value of the ratio of the natural frequencies of the damper mass and the engine mass, determined by relation (7), is substituted into the expression for the dynamic amplification factor of the engine mass (4), and the first derivative of the resulting function with respect to the variable  $\beta^2$  is set to zero, two values of the dimensionless damping

coefficient are obtained at which the dynamic amplification factor of the engine mass reaches its maximum value. These two values of the dimensionless damping coefficient are:

$$\beta_{(1)}^2 = \frac{\mu \left( 3 - \sqrt{\frac{\mu}{\mu+2}} \right)}{8(1+\mu)}, \quad (8)$$

$$\beta_{(2)}^2 = \frac{\mu \left( 3 + \sqrt{\frac{\mu}{\mu+2}} \right)}{8(1+\mu)},$$

It is recommended, as can be found in reference [4], that the value of the internal damping of the torsional vibration damper be taken as the average value from (8), so the optimal value of the dimensionless internal damping coefficient of the torsional damper is:

$$\beta_{opt}^2 = \frac{3\mu}{8(1+\mu)}, \quad (9)$$

By applying the torsional vibration damper for the crankshaft described here, it is possible to reduce the oscillation amplitudes only at a specific oscillation frequency (at one critical speed). This is because the entire equivalent torsional oscillatory system of the engine is represented by only one concentrated mass, so such a system has only one degree of freedom (one natural frequency). Therefore, the damper is designed to reduce the amplitude of oscillations as much as possible during resonant oscillation (one resonant regime due to one degree of freedom). Since the real torsional oscillatory system of an internal combustion engine is a system with multiple degrees of freedom, and thus has several natural frequencies of oscillation, the damping of torsional oscillations is usually chosen to be applied at the resonant regime where the oscillation amplitudes have the maximum value. It is expected that due to the extremely high damping in the damper itself, the amplitudes at other resonant regimes will also be lower than in the case when there is no torsional damper. If this is not the case, it is necessary to apply another model for calculating the torsional vibration damper for the crankshaft.

The moment of inertia of the concentrated mass representing the effect of the entire engine, respecting the principle of energy equality, is obtained as the sum of the moments of inertia of all masses in the equivalent torsional oscillatory system without the torsional vibration damper. The natural frequency of the system with one mass is equal to the frequency of the resonant harmonic at the speed where the oscillation amplitudes have the maximum value, in this case, the 3rd harmonic at 1075 rpm. With such a defined system, where the entire engine is represented by one concentrated mass, and by applying the described torsional damper model, Figures 17 and 18 show the effect of the damper on the dynamic amplification factor of the engine mass, as well as the damper mass.

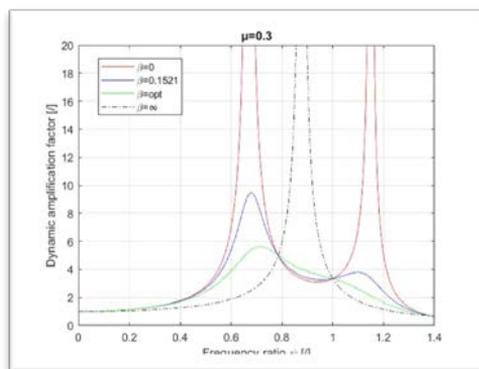
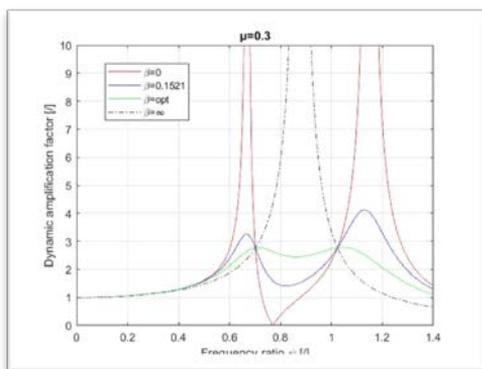


Figure 17 Dynamic amplification factor of the engine mass as a function of the frequency ratio for the given mass ratio

Figure 18 Dynamic amplification factor of the damper mass as a function of the frequency ratio for the given mass ratio

From the above figures, it is clearly visible that there are two frequency ratios (the ratio of the excitation frequency to the natural frequency of the engine mass) at which the dynamic amplification factor is independent of damping. These are the two points on the diagrams where all the curves representing different values of the dimensionless damping coefficient intersect. It can also be observed that the dynamic amplification factor is smallest when the damping in the damper is optimal.

By applying the described model of the torsional vibration damper, diagrams like those shown in Figures 19, 20, and 21 can be generated for different values of the mass ratio, i.e., the moments of inertia of the damper and the engine. Based on these diagrams, certain conclusions can be drawn regarding the design of the torsional vibration damper. From the diagram in Figure 19, it is clear that as the moment of inertia of the damper mass increases, with the engine's moment of inertia considered constant, the dynamic amplification factor of the engine mass decreases, meaning the oscillation amplitudes will be smaller, provided the damping is at an optimal level. On the other hand, optimal damping increases with the increase in the moment of inertia of the damper mass (Figure 20), while the difference between the dynamic amplification factors of the damper mass and the engine mass decreases with the increase in the moment of inertia of the damper mass (Figure 21). Based on the above, it is concluded that to achieve the greatest possible reduction in oscillation amplitudes, it is necessary for the moment of inertia of the damper to be as large as possible. Additionally, the stresses on the damper's damping element will be lower as the moment of inertia of the damper mass increases, due to the smaller difference in the dynamic amplification factors of the two masses (Figure 21). However, the moment of inertia of the damper mass cannot be arbitrarily large. The increase in moment of inertia is achieved either by increasing the mass or the diameter of an element. In this specific case, increasing the mass of the damper's inertial mass would lead to additional bending stresses on the crankshaft, which is not problematic because the crankshaft endures much higher stresses during operation. On the other hand, increasing the diameter of the damper's inertial mass is limited by the available space in the engine compartment. Furthermore, a large moment of inertia of the damper mass requires higher optimal damping, which complicates and increases the cost of constructing the damping element itself. Due to all of the above, the design of the torsional vibration damper must be a compromise between opposing requirements.

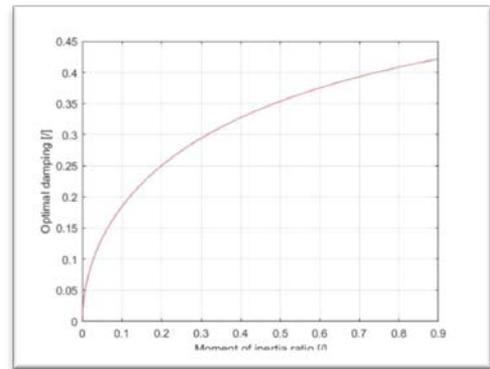
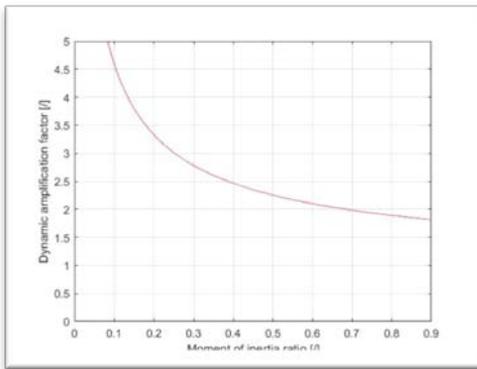


Figure 19 Dependence of the dynamic amplification factor of the engine mass on the ratio of moments of inertia

Figure 20 Dependence of optimal damping in the damper on the ratio of moments of inertia

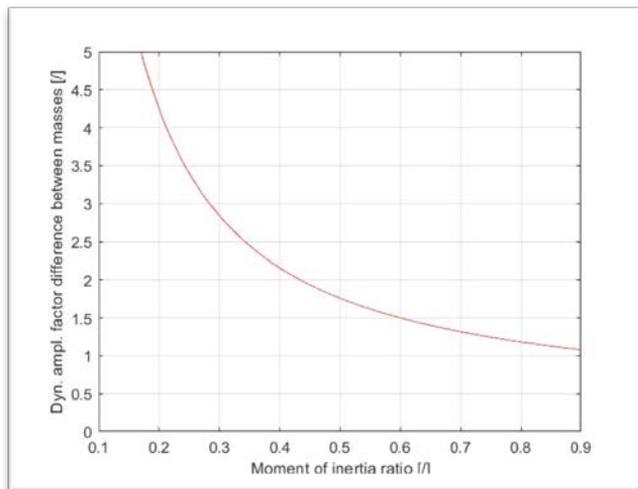


Figure 21 Difference in dynamic amplification factors between the engine mass and the damper mass as a function of the ratio of moments of inertia

For the engine that is the subject of this study, it was decided that the ratio of the moments of inertia of the damper mass and the engine would be 0.3. With this selected ratio of moments of inertia, the optimal damping coefficient of the damping element of the torsional damper was determined using formula (9). After defining the ratio of moments of inertia and the optimal damping coefficient, it is necessary to perform the calculation of torsional oscillations on the new equivalent torsional oscillatory system (with the damper), which now has one additional concentrated mass (the damper mass) and one additional section (the damper's damping element). The resulting amplitudes of all masses and stresses in all sections of such an equivalent torsional oscillatory system are shown in Figures 22 and 23.

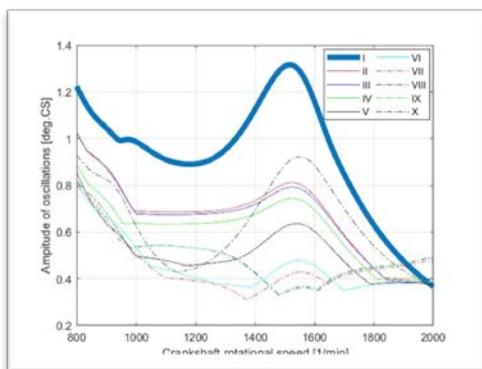


Figure 22 Oscillation amplitudes of the oscillatory system with the damper as a function of engine speed

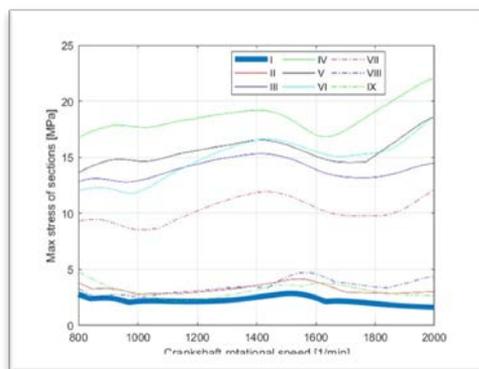


Figure 23 Stresses in sections of the oscillatory system with the damper as a function of engine speed

## 5 CONCLUSION

Based on the conducted calculation of the torsional oscillations of the crankshaft of the “V-46-6” engine, which does not have a torsional vibration damper, it has been determined that the oscillation amplitudes are extremely large, even exceeding  $2^\circ$  in some regimes (Figure 10). Such high values of oscillation amplitudes inevitably lead to increased vibrations of the entire engine, and additionally, the supplementary stresses on the crankshaft due to torsional oscillations are above the permissible limit (Figure 11). Besides the purely mechanical problems resulting from the torsional oscillations of the crankshaft, they also cause certain issues that affect the engine’s working cycle. Given that the amplitude of the mass representing the auxiliary drive is extremely large, it is likely that the process of timely opening and closing of the valves, i.e., the working fluid exchange process, is compromised. As a result, the engine’s power is reduced, and its efficiency worsens. Additionally, due to the large oscillation amplitudes, the fuel injection process becomes deregulated because the high-pressure pump is driven by the crankshaft. This fact also leads to a decrease in power and a deterioration in efficiency. To mitigate all the aforementioned negative consequences of the intense torsional oscillations of the crankshaft as much as possible, it was decided to install a torsional vibration damper on the crankshaft.

The described model of the torsional vibration damper is the simplest model, assuming that the damping element has a linear characteristic, i.e., that its torsional stiffness and damping coefficient are constant values. This assumption is rather crude and does not correspond to the real oscillation process because both torsional stiffness and damping coefficient are variables that depend on the oscillation angle (in dampers with a rubber damping element, which are most commonly used).

The installation of the torsional vibration damper according to the described model successfully reduced both the amplitudes and the stresses on the crankshaft. However, due to the assumption of the torsional vibration damper as a linear element, the obtained results need to be experimentally verified. Despite this, the results presented here provide a good starting point for the design of a torsional vibration damper.

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