



TRAFFIC SIGNAL OPTIMIZATION AT A QUADRATIC ROAD INTERSECTION USING A SWARM INTELLIGENCE ALGORITHM

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ABSTRACT: Quadratic Road Intersection (QRI) belongs to the group of traffic intersections with alternative designs, which have become popular due to their ability to alleviate traffic congestion and reduce travel time. A common feature of all alternative intersection designs is their implementation in suburbs, since they require more space compared to traditional intersections. The second common characteristic of the family of alternative intersections is treating left-turning vehicles in an unconventional way. QRI eliminates all left-turn movements at the main intersection, using quadratic bypasses to accommodate the missing turns. This creates two secondary intersections downstream from the main one, typically controlled by traffic lights. There are two phasing schemes that are used to control QRI: with one controller (Reid phases) and with three controllers, one for each intersection. The latter assumes optimization of traffic progression between the separated intersections that the QRI consists of. In this study, the QRI layout from the field is used to investigate control delay discrepancies in the implementation of the two phasing schemes. Moreover, this study develops a unique offset optimization approach, especially suited for QRI. In that endeavour, a swarm optimization technique, Bee Colony Optimization (BCO), is leveraged to optimize traffic control parameters: cycle, splits, and offsets. BCO is a well-known metaheuristic approach, based on the natural process of collecting nectar by honeybees. BCO has already shown high performance in traffic control optimization tasks. Although more complex to implement, the results show that the three-controller phase scheme, with offset optimization, outperforms the Reid phase scheme in terms of control delay as the primary performance measure in traffic signal control optimization problems.

KEY WORDS: Quadratic road intersection, Reid phase scheme, offset optimization, bee colony Optimization, control delays

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OPTIMIZACIJA SVETLOSNIH SIGNALA NA KVADRATNOJ RASKRSNICI PRIMENOM INTIGENCIJE ROJA

REZIME: Kvadratna raskrsnica (QRI) pripada grupi saobraćajnih raskrsnica sa alternativnim dizajnom, koje su postale popularne zbog svoje sposobnosti da ublaže saobraćajna zagušenja i smanje vreme putovanja. Zajednička karakteristika svih alternativnih raskrsnica je njihova primena u prigradskim područjima, zbog činjenice da zahtevaju više prostora u poređenju sa tradicionalnim raskrsnicama. Druga zajednička karakteristika ovih nekonvencionalnih saobraćajnih rešenja je tretiranje vozila koja skreću levo na alternativan način. QRI eliminiše sva leva skretanja na glavnoj raskrsnici, koristeći kvadratne obilazne saobraćajne trake kako bi se opslužila vozila u levom skretanju. Na taj način nastaju dve sekundarne raskrsnice nizvodno od glavne, koje su obično kontrolisane svetlosnim signalima. Postoje dva plana faza koje se koriste za upravljanje QRI: sa jednim kontrolerom (Reid faze) i sa tri kontrolera, po jednim za svaku raskrsnicu. Ova druga podrazumeva optimizaciju progresije saobraćaja između odvojenih raskrsnica od kojih se QRI sastoji. U ovom istraživanju koristi se QRI geometrija sa terena kako bi se ispitala razlike u vremenskim gubicima vozila pri primeni predmetnih planova faza. Pored toga, u radu je razvijen jedinstveni pristup optimizacije pomaka zelenog vremena, posebno prilagođen QRI raskrsnicama. Za optimizaciju parametara upravljanja svetlosnih signala: ciklusa, raspodele zelenog vremena i pomaka zelenog vremena, koristi se algoritma iz klase optimizacije roja: optimizacija kolonijom pčela (BCO), dobro poznata metaheuristička metoda zasnovana na prirodnom procesu prikupljanja nektara kod pčela. BCO je pokazao visoke performanse u zadacima optimizacije upravljanja saobraćajem. Iako je složenija za implementaciju, rezultati pokazuju da plan faza sa tri kontrolera, uz optimizaciju pomaka zelenog vremena, daje bolje rezultate od Reid faza u pogledu vremenskih gubitaka vozila, kao primarne mere performansi u problemima optimizacije upravljanja svetlosnim signalima.

KLJUČNE REČI: *Kvadratna raskrsnica, Reid faze, optimizacija pomaka zelenog vremena, optimizacija kolonijom pčela, vremenski gubici vozila*

TRAFFIC SIGNAL OPTIMIZATION AT A QUADRATIC ROAD INTERSECTION USING A SWARM INTELLIGENCE ALGORITHM

Aleksandar Jovanović

INTRODUCTION

Traffic engineers, researchers, and practitioners seek solutions that will mitigate the global problem of traffic congestion, which causes emissions and urban contamination. Among all approaches to this problem, intersections with Alternative Geometrical Design (AGD) arose as a practical solution that improves traffic performance measures such as vehicle delay and travel time [5]. AGDs are typical suburban facilities, where urban streets meet rural highways. The reason for this lies in the space requirements and the specific demand patterns that suit these intersections. For instance, some types of AGD fit well with high through traffic demands on the main road and relatively low demands on the side streets. On the other hand, Quadratic Road Intersection (QRI) might accommodate relatively high traffic demands from both the main and side roads, which makes it suitable for implementation in inner parts of cities.

QRI eliminates left-turn movements from the main intersection and redirects them toward two downstream intersections, which are connected by a quadratic bypass road. At first glance, it might look that vehicles travel more compared to the traditional layout. However, by significantly improving the capacity of the main intersection, studies [13] show improvement in overall travel time and vehicle delays.

Due to relatively simplified phase plans, fixed-time traffic control is the preferred way to control traffic demands at QRI. However, it remains an open problem whether it is preferable to use a single controller for all three intersections that the QRI consists of, or to implement a separate controller for each intersection. In the first case, the so-called Reid phase plan was developed [12], and it is recommended in the case of shorter distances between the three intersections and unbalanced traffic demands on the main intersection. On the other hand, when intersections are farther apart and traffic demands on the main intersection are relatively balanced between the two directions, the solution with three controllers is preferable [13].

The aim of this study is to investigate these recommendations under different traffic demand patterns using the geometrical layout of a field QRI. The first objective is to optimize traffic signal parameters at QRI to minimize vehicle delay using both Reid phases and the three-controller phase scheme. The second objective is to develop offset optimization especially suitable for implementation in QRI.

As the optimization tool, the Bee Colony Optimization (BCO) technique is leveraged for obtaining control parameters: cycle, splits, and offsets. BCO is inspired by bee behaviour in nature during the process of nectar collection. The first mathematical formulation and application of BCO were proposed by [10]. Since then, BCO has been shown to be a reliable tool for traffic control optimization at signalized intersections [7,9]. Also, BCO has already been applied to the optimization of AGD [8].

In recent decades, QRI has attracted moderate research interest. In his study, Reid (2000) found that QRI lowers overall travel time by 22% compared to the traditional intersection layout. At the same time, travel time for left-turning vehicles is not significantly changed. The Federal Highway Administration (FHWA) provides a detailed guide on QRI, including recommendations for geometry, markings, and pedestrian accommodation [5]. During the

construction of QRI in Fairfield, Ohio, the project team found that the implemented QRI had a positive influence on the environment and the economy of the community [1].

The literature review opens a vast space for research on QRI, especially in the area of traffic signal control optimization. The contribution of this study is to examine two recommended phase plans on a QRI from the field under different traffic demand patterns. In addition, a unique set of equations is developed for offset optimization in the case of the three-controller phase scheme.

After the introduction, the second section presents the problem statement. The third section is dedicated to the mathematical description of the problem and the proposed solution. The fourth section shows a numerical example with results. Finally, the fifth section concludes the study and outlines possible directions for future research.

1 PROBLEM STATEMENT

In this study the QRI layout from Fairfield, Ohio ($39^{\circ}19'26.16''$ N $84^{\circ}30'16.50''$ W), which opened in January 2012, is used. Figure 1 presents the geometry of subject QRI, with marked traffic movements (A, B... R). Right-turn movements, except for traffic movement H, are not controlled by traffic lights. Pedestrian demands are neglected.

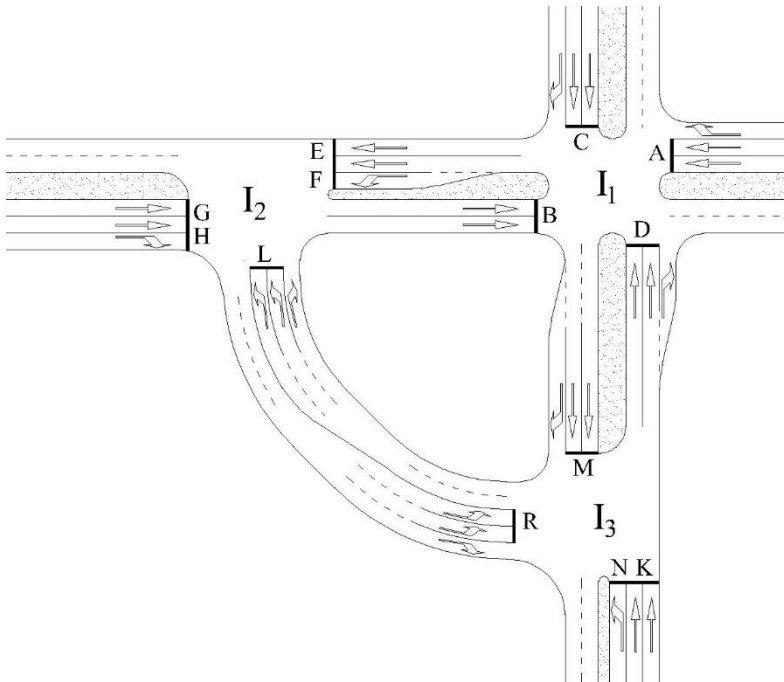


Figure 1. The graphical representation of QRI layout

Two possible signal phase schemes used to control QRI are presented in Figure 2. In the upper part of Figure 2, the Reid scheme is shown, while the lower part presents the signal phase scheme with three controllers. In the latter case, the main intersection is controlled by two phases, while the two downstream intersections are each controlled by three phases.

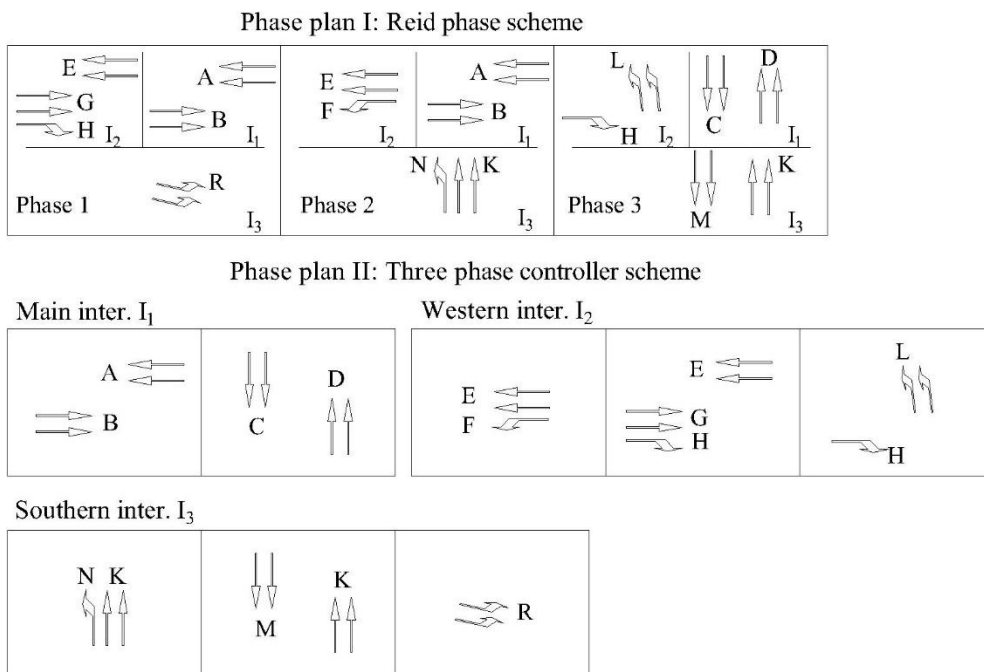


Figure 2. Two phases' planes for QRI

Because right-turning vehicles are served without traffic light control, the extra travel time for left-turning vehicles will be negligible. However, this is not the case for left-turning vehicles in movements R and B. Approximately, the extra travel time is equal to the double travel time between intersections (t_{tr}). Because delay is the main performance measure in this paper, these additional travel times will not affect the optimization results and will not be considered in the mathematical model.

Engineering questions that arise from the problem statement are: (1) which signal plan should be applied considering different traffic demand patterns and the distances between the three intersections that the QRI consists of; (2) for the applied signal plan, what should be the values of the control parameters: cycle, splits, and, in the case of signal plan 2, offsets; (3) how to set up the offset optimization for this unique node of three intersections.

To compare these differences, vehicle delay is used as the traditional performance measure in the case of unsaturated traffic conditions. Because QRI is designed for high traffic demands, oversaturated traffic conditions are not expected. Therefore, this study does not consider traffic congestion at QRI.

2 MATHEMATICAL FORMULATION AND PROPOSED SOLUTION

According to the QRI layout showed in Figure 1, let us define by M the set of traffic movements, where m is the traffic movement index, $m = A, B, \dots R$. Let us define the vehicle delay (d_m) of m -th movement in the following way [4]:

$$d_m = d_{1m} \cdot (PF_m) + d_{2m} \quad \forall m \in M \quad (1)$$

where d_{1m} – uniform delay in the m -th movement (s/veh), which can be calculated as:

$$d_{1m} = \frac{0.5 \cdot C \cdot \left(1 - \frac{g_m}{C}\right)^2}{1 - \left[\min(1, X_m) \cdot \frac{g_m}{C}\right]} \quad (2)$$

where:

C – cycle length (s),

g_m – green time in the m -th movement (s),

X_m – volume-to-capacity ratio of the m -th group, which can be calculated as:

$$X_m = \frac{q_m/s_m}{g_m/C} \quad (3)$$

where:

q_m – traffic demand in the m -th movement (veh/h),

s_m – saturation flow rate in the m -th movement (veh/h),

d_2 – incremental delay in the m -th movement (s/veh), which can be calculated as:

$$d_{2m} = 900T \cdot \left[(X_m - 1) + \sqrt{(X_m - 1)^2 + \frac{4 \cdot X_m}{c_m \cdot T}} \right] \quad (4)$$

where:

T – analysis time (1 h),

c_m – capacity in the m -th movement, can be calculated as:

$$c_m = s_m \cdot \frac{g_m}{C} \quad (5)$$

Progression factor (PF) can take values between 0 and 1, where 0 means that the whole platoon of vehicles faces red at the downstream intersection, while 1 means that the whole platoon passes through green at the downstream intersection. PF is calculated based on the formulas presented in the study by [2].

PF_m – progression factor in the m -th movement, which can be calculated as:

$$PF_m = 1 - \left\{ \frac{1}{B_m} \left[\max(0, \min(\tilde{r}_m + B_m, g_{down\ m}) - \tilde{r}_m) + \max(0, \min(\tilde{r}_m + B_m - C_m, g_{down\ m})) \right] \right\} \quad (6)$$

where:

B_m – the platoon bandwidth. It is assumed to be equal to the green time of the upstream intersection for movement m ,

$g_{down\ m}$ – the green time of the downstream intersection for movement m ,

$\tilde{r}_m$ – the relative arrival time of the beginning of the platoon at the downstream intersection for movement m , measured within the signal cycle, which can be calculated as:

$$\tilde{r}_m = (t_{tr} - \theta_m) \bmod C \quad (7)$$

where θ_m is the offset for the m -th movement. The offset is the time difference between phase initiation at the upstream and downstream intersections. It affects the movement at the downstream intersections.

Let us define I as the set of all intersections that QRI consist of, where i is the intersection index, $i = 1, 2, 3$. In a similar way, let us define P as the set of phases that control i -th intersection, where p is the phase index, $p = 1, 2, 3$. For instance, considering phase plan II from Figure 2, offset θ_{31-23} represents the time difference between the initiation of phase 1 at intersection 3 and phase 3 at intersection 2. Because it affects movement L, it will be denoted as θ_L .

In the case of phase plan I from Figure 2, the mathematical formulation of the problem is defined as:

$$\min \frac{\sum_{m=1}^{|M|} d_m \cdot q_m}{\sum_{m=1}^{|M|} q_m} \quad (8)$$

subject to:

$$C_{min} \leq C \leq C_{max} \quad (9)$$

$$g_{p \min} \leq g_p \leq g_{p \max}, \quad \forall p \in P \quad (10)$$

$$\sum_{p=1}^{|P|} g_p + L = C \quad (11)$$

$$C, g_p \text{ and } L \in Z_{>0} \quad (12)$$

where:

g_p – the green time of the p -th phase (s),

L – the cycle lost time at the QRI (s).

Equation (8) presents the total average vehicle delay at the QRI, which needs to be minimized. In this case, PF is equal to 1 for all traffic movements. Equation (9) sets the cycle length between minimum and maximum values, while Equation (10) does the same for each control phase. Equation (11) defines the relation between green time, cycle lost time, and cycle length.

In the case of phase plan II from Figure 2, the mathematical formulation of the problem is defined as:

$$\min \frac{\sum_{m=1}^{|M|} d_m \cdot q_m}{\sum_{m=1}^{|M|} q_m} \quad (13)$$

subject to:

$$C_{min} \leq C \leq C_{max} \quad (14)$$

$$g_{ip \min} \leq g_{ip} \leq g_{ip \max}, \quad \forall p \in P, \forall i \in I \quad (15)$$

$$\sum_{p=1}^{|P|} g_{ip} + L_i = C, \quad \forall i \in I \quad (16)$$

$$\theta_D + \theta_M = n \cdot C + g_{31} + (\Delta t + y) \quad (17)$$

$$\theta_B + \theta_E = n \cdot C - g_{21} - (\Delta t + y) \quad (18)$$

$$\theta_R + \theta_L = n \cdot C - g_{33} + g_{22} \quad (19)$$

$$0 \leq \theta_m \leq C, \quad \forall m \in M \quad (20)$$

$$C, g_{ip} \text{ and } L_i \in Z_{>0} \quad (21)$$

$$\theta_m \text{ and } n \in Z_{\geq 0} \quad (22)$$

where:

g_{ip} – the green time of the p -th phase at the i -th intersection (s),

L_i – the cycle lost time at the i -th intersection (s),

$\Delta t + y$ – all-red time plus yellow time. In this study, this value is equal to 5 s in all cases.

Equations (17–22) are developed in accordance with area traffic control principles. More details can be found in the papers by [5,6].

The mathematical optimization is conducted using the BCO metaheuristic. The pseudocode of the BCO algorithm is presented below [11]:

Pseudocode for BCO

Set number of bees: B

Set the number of iterations: IT

Set the number of forward and backward passes in a single iteration: NP

Set number of changes: NC

REPEAT

 For $i = 1$ to B

 Determine an initial solution for the i th bee

 Evaluate the solution of the i th bee

 Set S : calculate the best solution of the bees

 For $j = 1$ to IT :

 For $i = 1$ to B :

 Set the initial solution for bee i

 For $k = 1$ to NP :

 For $i = 1$ to B :

 For $r = 1$ to NC :

 Evaluate modified solutions generated by possible changes of the i th bee solution

 By roulette wheel selection choose one of the modified solutions

 For $i = 1$ to B :

 Evaluate solution of the i th bee

 For $i = 1$ to B :

 Decide whether the i th bee is loyal

 For $i = 1$ to B :

 if the bee i is not loyal

 Choose one of the loyal bees to be followed by the i th bee

 if the best solution of the bees is better than the solution S

 Set S : the best solution of the bees

UNTIL criterion is met

RETURN S

More details about the BCO application and the optimization parameter settings in traffic signal timing control problems can be found in [6].

3 A NUMERICAL EXAMPLE AND RESULTS

Fixed-time optimization, using BCO, is conducted on the geometrical layout presented in Figure 1, with the following parameters:

- Travel time between all intersections is 14 seconds ($t_{tr} = 14$ s). The travel time (t_{tr}) between all intersections is assumed to be the same.
- Minimum and maximum cycle values are 30 and 120 seconds, respectively ($C_{min} = 30$ s; $C_{max} = 120$ s),
- Minimum and maximum green time values are 5 and 70 seconds, respectively ($g_{p_{min}} = 5$ s; $g_{p_{max}} = 70$ s),
- Cycle lost time is 15 seconds ($L = 15$ s), except in the case of phase plan II for the main intersection, where $L = 10$ s.

Based on prior experience, the following BCO parameters were adopted:

- Number of bees involved in the search: 20,
- Number of forward/backward passes in a single BCO iteration: 20,
- Number of changes in one forward pass: 1, and
- CPU time allocated to artificial bees to search for the solutions (in all tests): 6 min.

The base traffic demand pattern (TD #1 in veh/h) is adopted from the FHWA report on QRI (Hughes et al., 2010), while saturation flow rates (s in veh/h) are calculated according to HCM (2010) and presented in Table 1. Additionally, four more traffic demand patterns (TD #2-5) were developed to address the questions stated in the problem statement. Finally, to investigate the influence of different geometries on QRI performance, the travel time between intersections is changed to 7 s and 21 s under all traffic demand scenarios.

Table 1. Traffic demands and saturation flow rates

q (veh/h)	A	B	C	D	E	F	G	H	L	K	N	M	R
TD #1	750	770	750	850	800	150	620	500	160	650	160	600	200
TD #2	750	700	450	450	800	150	620	450	160	300	160	370	150
TD #3	1050	1070	750	850	1000	250	920	500	160	650	160	600	200
TD #4	750	1070	750	850	800	150	920	700	160	650	160	600	250
TD #5	450	400	450	450	550	100	320	450	160	300	160	370	150
s (veh/h)	3800	3800	3800	3800	3800	1500	3800	1500	2800	3800	1500	3800	2800

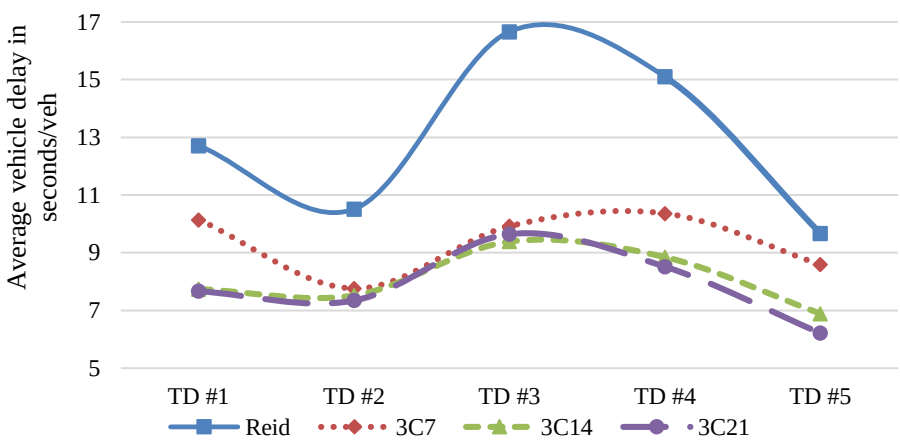
All traffic scenarios (from TD #2 to TD #5) deviate from TD #1 to capture fluctuations in traffic demand within the QRI. TD #2 represents a decrease in demand in one direction at the main intersection, while TD #3 represents an increase in demand in one direction of the main intersection. TD #4 reflects an increase in traffic demand at the west intersection, whereas TD #5 represents a decrease in demand in both directions at the main intersection.

Table 2 shows the optimization results in terms of control delay as the primary performance measure. Furthermore, for the base traffic demand pattern, the lower part of Table 2 presents the corresponding control parameters.

Table 2. The results of average vehicle delays (in seconds/veh)

t _r = 14 s														
TD #				1		2		3		4		5		
Reid phase				12.71		10.51		16.66		15.1		9.67		
The three phase controller				7.74		7.52		9.39		8.85		6.89		
The three-phase controller (the average delay values in seconds/veh)														
TD #1		TD #2			TD #3			TD #4			TD #5			
t _r = 7 s	t _r = 21 s	t _r = 7 s	t _r = 21 s	t _r = 7 s	t _r = 21 s	t _r = 7 s	t _r = 21 s	t _r = 7 s	t _r = 21 s	t _r = 7 s	t _r = 21 s	t _r = 7 s	t _r = 21 s	
10.14	7.67	7.76	7.35	9.92	9.65	10.36	8.52	8.59	6.22					
Intersection control parameters (in seconds). For Reid phase: C=46 s, g ₁ =10 s, g ₂ =7 s, g ₃ =14 s														
C	g ₁₁	g ₁₂	g ₂₁	g ₂₂	g ₂₃	g ₃₁	g ₃₂	g ₃₃	θ _D	θ _M	θ _B	θ _E	θ _R	θ _L
40 s	19 s	11 s	6 s	14 s	5 s	8 s	11 s	6 s	13 s	40 s	10 s	19 s	19 s	29 s

Considering the results, one might consider the three-phase controller to be more suitable for the QRI from the perspective of average vehicle delays. For the base traffic demand pattern and travel time from the field of 14 s, the three-phase controller outperformed the Reid phase by 39.1%. For TD #2, TD #3, TD #4, and TD #5, the three-phase controller performed better than the Reid phase by 28.45%, 43.64%, 41.39%, and 28.75%, respectively. Figure 3 presents a graphical representation of the results.



Legend: 3C – The three-phase controller; 7, 14, 21 – seconds of travel time

Figure 3. The results

By changing the travel time, it is shown that Reid phases perform better for shorter distances between the intersections that form the QRI compared to longer distances. However, with properly optimized offsets, the three-phase controller is still the preferable choice.

CONCLUSIONS

This paper addresses the problem of fixed traffic signal control optimization at QRI, an alternative intersection design that has gained popularity in the USA and has potential for implementation in other parts of the world. BCO, a well-known swarm-based metaheuristic, is used as the optimization technique. Two recommended phase plans are examined: one

that assumes a single controller, the so-called Reid phase scheme, and another with three controllers and offset optimization.

It has been shown that, with proper optimization of offsets, the three-controller phase plan outperforms the Reid phase in all traffic scenarios and for all tested changes in travel time. It has also been noted that the Reid signal plan provides acceptable results when the distances between intersections are shorter, although the three-controller solution remains the preferable option. In addition, the contribution of this paper lies in the formulation of offset optimization equations that are unique for the QRI.

In future research, travel time, number of stops, and ecological parameters could be considered during fixed-time optimization. In addition, it should be examined whether real-time traffic control algorithms can further improve performance measures at QRI.

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